How Quantitative Methods Can Help To Understand Some Asset Management Problems?\textsuperscript{1}

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\textsuperscript{1}The opinions expressed in this presentation are those of the author and are not meant to represent the opinions or official positions of Lyxor Asset Management.
Outline

1. Passive Management and Market-cap Indexation
2. Rationale of Diversified Funds
3. Weights Constraints and Portfolio Theory
4. Volatility Costs in a Trend-following Strategy
5. Understanding Strategic Asset Allocation
The Problem

An index $I_t$ at time $t$ is defined by:

$$I_t = \sum_{i=1}^{n} w_{i,t} P_{i,t}$$

where $w_{i,t}$ and $P_{i,t}$ is the weight and the price of the $i^{th}$ asset at date $t$.

We are interested in two types of weights:

- Weights can depend on prices:
  $$w_{i,t} = f(P_{i,t})$$

- Weights are not linked to prices:
  $$w_{i,t} \perp P_{i,t}$$
Some weighting schemes are not good

If we suppose that we have \( w_{i,t} = \bar{w}_{i,t} P_{i,t} \), we obtain:

\[
l_t = \sum_{i=1}^{n} \bar{w}_{i,t} P_{i,t}^2
\]

If prices are log-normal distributed, what is the distribution of \( l_t \) ?
Pros and Cons of Market-cap Indexation

Pros of market-cap indexation

- A convenient and **recognized approach** to participate to broad equity markets.
- **Management simplicity**: low turnover & transaction costs.

Cons of market-cap indexation

- Trend-following strategy: momentum bias leads to bubble risk exposure as weight of best performers ever increases.
  ⇒ Mid 2007, financial stocks represent 40% of the Eurostoxx 50 index.
- Growth bias as high valuation multiples stocks weight more than low-multiple stocks with equivalent realised earnings.
  ⇒ Mid 2000, the 8 stocks of the technology/telecom sectors represent 35% of the Eurostoxx 50 index.
  ⇒ 2½ years later after the dot.com bubble, these two sectors represent 12%.
- Concentrated portfolios.
  ⇒ The top 100 market caps of the S&P 500 account for around 70%.
- Lack of risk diversification and high drawdown risk: no portfolio construction rules leads to concentration issues (e.g. sectors, stocks).
Statistical Measures of Concentration

- The Lorenz curve $L(x)$
  It is a graphical representation of the concentration. It represents the cumulative weight of the first $x\%$ most representative stocks.

- The Gini coefficient
  It is a dispersion measure based on the Lorenz curve:

  $$G = \frac{A}{A+B} = 2 \int_0^1 L(x) \, dx - 1$$

  $G$ takes the value 1 for a perfectly concentrated portfolio and 0 for the equally-weighted portfolio.
### Concentration of Equity Indexes (December 31, 2009)

<table>
<thead>
<tr>
<th>Index</th>
<th>Gini</th>
<th>10</th>
<th>25</th>
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<td>0.27</td>
<td>23</td>
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<td>HSCEI</td>
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<td>KOSPI</td>
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<td>94</td>
<td>98</td>
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</tbody>
</table>

(*) In the case of the SX5P Index, 10% of stocks (respectively 25% and 50%) represent 23% of weight in the index (respectively 45% and 68%).
Main argument of passive management:

**The Market Cap Index = The Tangency Portfolio**

In the modern portfolio theory of Markowitz, we maximize the expected return for a given level of volatility:

$$\max \mu(w) = \mu^\top w \quad \text{u.c.} \quad \sigma(w) = \sqrt{w^\top \Sigma w} = \sigma^*$$

- The optimal portfolio is the tangency portfolio.
- Main problem: the solution is very sensitive to the vector of expected returns ⇒ the solution is not robust.
- If the market cap index is the optimal portfolio, it means that expected returns are persistent.

- Academic research has illustrated that Capitalization-weighted indexes are not tangency portfolios.
- Dynamics of cap-weighted indexes = dynamics of price-weighted indexes (e.g. Nikkei and Topix indexes).
Alternative-weighted indexation aims at building passive indexes where the weights are not based on market capitalization.

Two sets of responses:

1. Fundamental indexation ⇒ promising *alpha*.
2. Risk-based indexation ⇒ promising *diversification*.

Two ways of using risk-based indexation:

1. Substitute as the capitalized-weighted index.
2. Complement to the capitalized-weighted index.
Portfolio Construction

Equally-weighted (1/n)
Most Diversified Portfolio (MDP)

Minimum-variance (MV)
Equal-Risk Contribution (ERC)

Notations
Let \( w \) be the vector of weights, \( \mu \) the vector of risk premia (e.g. expected returns) and \( \Sigma \) the covariance matrix of returns. The volatility of the portfolio is:

\[
\sigma (w) = \sqrt{w^\top \Sigma w}
\]

wheras its expected return is:

\[
\mu (w) = w^\top \mu
\]
We have:

\[ w_i = \frac{1}{n} \]

Some properties

- It is the less concentrated portfolio:
  \[ G_w = 0 \]

- It is a contrarian strategy.

- It has a take-profit scheme.
The Minimum-Variance Portfolio

The problem is: \( w^* = \arg \min \sqrt{w^\top \Sigma w} \text{ u.c. } 1^\top x = 1 \text{ and } 0 \leq x \leq 1. \)

In the short-selling case, the lagrangian function is:

\[
f(w; \lambda_0) = \sigma(w) - \lambda_0 (1^\top w - 1)
\]

The solution \( w^* \) verifies the following system of first-order conditions:

\[
\begin{align*}
\partial_x f(w; \lambda_0) &= \frac{\partial \sigma(w)}{\partial w} - \lambda_0 1 = 0 \\
\partial_{\lambda_0} f(w; \lambda_0) &= 1^\top w - 1 = 0
\end{align*}
\]

We have:

\[
\frac{\partial \sigma(w)}{\partial w_i} = \frac{\partial \sigma(w)}{\partial w_j} = \sigma(w) \text{ for all } i, j
\]

In the case of no-short selling, write the Kühn-Tucker conditions and we have:

\[
\begin{align*}
\frac{\partial \sigma(w)}{\partial w_i} &= \frac{\partial \sigma(w)}{\partial w_j} \text{ for all } w_i \neq 0, w_j \neq 0
\end{align*}
\]
The MDP/MSR Portfolio

Let $D(w)$ be the diversification ratio:

$$D(w) = \frac{\sqrt{w^\top \tilde{\Sigma} w}}{\sqrt{w^\top \Sigma w}} = \frac{w^\top \sigma}{\sqrt{w^\top \Sigma w}}$$

where $\tilde{\Sigma}$ is the covariance matrix with $\tilde{\Sigma}_{i,j} = \sigma_i \sigma_j$ (all the correlations are equal to one). We have $D(w) \geq 1$. The MDP portfolio is defined by:

$$w^* = \arg\max_{u.c. \; 1^\top x = 1 \text{ and } 0 \leq x \leq 1} D(w)$$

Remark

If we assume that the Sharpe ratio is the same for all the assets $-\mu_j - r = s \times \sigma_j$, we obtain:

$$sh(w) = \frac{w^\top \mu - r}{\sqrt{w^\top \Sigma w}} = s \times D(w)$$

Maximizing $D(w)$ is equivalent to maximize $sh(w)$. 
The ERC Portfolio

The Euler decomposition gives us:

\[ \sigma(w) = \sum_{i=1}^{n} w_i \times \frac{\partial \sigma(w)}{\partial w_i} = \sum_{i=1}^{n} \text{RC}_i \]

The idea of the ERC strategy is to find a risk-balanced portfolio such that the risk contribution is the same for all assets of the portfolio:

\[ \text{RC}_i = \text{RC}_j \quad \text{for all } i,j \]
An example

3 assets.
Volatilities are respectively 20%, 30% and 15%.
Correlations are set to 60% between the first and second asset, and 10% for the third assets.

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<th>w_{i}</th>
<th>MR_{i}</th>
<th>RC_{i}</th>
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<td>18.8%</td>
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<td>8.6%</td>
<td>60.0%</td>
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<td>13.2%</td>
<td>21.7%</td>
<td>2.9%</td>
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<td>38.3%</td>
<td>7.5%</td>
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<td>14.3%</td>
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<td>4.6%</td>
<td>33.3%</td>
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<td>4.6%</td>
<td>33.3%</td>
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<tr>
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<td>49.3%</td>
<td>9.3%</td>
<td>4.6%</td>
<td>33.3%</td>
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<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td></td>
<td>13.8%</td>
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</table>
Comparison of the 4 Methods

**Equally-weighted (1/n)**
- Weights are equal
- Easy to understand
- Contrarian strategy with a take-profit scheme
- The least concentrated in terms of weights
- Do not depend on risks

**Minimum-variance (MV)**
- Low volatility portfolio
- The only optimal portfolio not depending on expected returns assumptions
- Good out of sample performance
- Concentrated portfolios
- Sensitive to the covariance matrix

**Most Diversified Portfolio (MDP)**
- Also known as the Max Sharpe Ratio (MSR) portfolio of EDHEC
- Based on the assumption that sharpe ratio is equal for all stocks
- It is the tangency portfolio if the previous assumption is verified
- Sensitive to the covariance matrix

**Equal-Risk Contribution (ERC)**
- Risk contributions are equal
- Highly diversified portfolios
- Less sensitive to the covariance matrix (than the MV and MDP portfolios)
- Not efficient for universe with a large number of stocks (equivalent to the 1/n portfolio)
Comparison of the 4 Methods

In terms of bets

\[
\begin{align*}
\exists i : w_i &= 0 \quad (\text{MV} - \text{MDP}) \\
\forall i : w_i \neq 0 \quad (1/n - \text{ERC})
\end{align*}
\]

In terms of risk factors

\[
\begin{align*}
w_i &= w_j \\
\frac{\partial \sigma(w)}{\partial w_i} &= \frac{\partial \sigma(w)}{\partial w_j} \quad (1/n) \\
\text{MV} \\

w_i \times \frac{\partial \sigma(w)}{\partial w_i} &= w_j \times \frac{\partial \sigma(w)}{\partial w_j} \quad (\text{ERC}) \\

\frac{1}{\sigma_i} \times \frac{\partial \sigma(w)}{\partial w_i} &= \frac{1}{\sigma_j} \times \frac{\partial \sigma(w)}{\partial w_j} \quad (\text{MDP})
\end{align*}
\]
Backtest with the DJ Eurostoxx 50 Universe

Backtesting rules

Monthly rebalancing of the weights.
The covariance matrix used for simulations is the empirical covariance matrix based on a rolling observation period of 1 year.
All indexes are price index (PI).
The study period is January 1993 – December 2009.

<table>
<thead>
<tr>
<th></th>
<th>CW</th>
<th>MV</th>
<th>ERC</th>
<th>MDP</th>
<th>1/n</th>
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<td>8.08</td>
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### Composition in % (January 2010)

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Total of components: 50

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</table>

Total of components: 50

### The Problem

#### Market-cap Indexation

#### Risk-Based Indexation

#### Backtest with the DJ Eurostoxx 50 Universe

**Rationale of Diversified Funds**
- Weights Constraints and Portfolio Theory
- Volatility Costs in a Trend-following Strategy

**Understanding Strategic Asset Allocation**

**Conclusion**

**Quantitative Methods in Asset Management**

**Thierry Roncalli**
Conclusion

- Risk-based indexation historically posts better risk-adjusted performance than capitalization-weighted indexation.
- It is a promising way for investors to gain access to a well-diversified and diversifying exposure (or beta) to broad equity markets.
- Some practical issues:
  1. Turnover managing;
  2. Market price impact minimizing;
  3. Transparency (passive indexation or active strategy?);
  4. Understanding the style bias (small caps, growth, sectors, etc.);
- Existence of professional solutions (indexes/mutual funds).
And Bonds?

- Market-cap indexation based on outstanding amount of debt.
- Ignores risk dimension (e.g. sovereign risk, country risk, etc.).

### Sovereign risk contribution of each state in the debt weighted index*

<table>
<thead>
<tr>
<th>Country</th>
<th>EGBI Weight</th>
<th>Risk contribution</th>
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<tbody>
<tr>
<td>Italy</td>
<td>23.5%</td>
<td>42.4%</td>
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<tr>
<td>Spain</td>
<td>9.7%</td>
<td>18.6%</td>
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<tr>
<td>France</td>
<td>22.2%</td>
<td>11.2%</td>
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<tr>
<td>Portugal</td>
<td>2.1%</td>
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</tr>
<tr>
<td>Ireland</td>
<td>2.1%</td>
<td>6.8%</td>
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<tr>
<td>Belgium</td>
<td>6.5%</td>
<td>4.7%</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>Greece</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Spain + Italy + Portugal + Ireland:

- **Spain**: 37.4%
- **Italy**: 76.0%

Index rebalancing, exit of Greece: November 2010

**Thierry Roncalli**

*Quantitative Methods in Asset Management*
The 3 Profiles

**Fund Profiles**
1. **Dynamic** (20% of bonds and 80% of equities)
2. **Balanced** (50% of bonds and 50% of equities)
3. **Defensive** (80% of bonds and 20% of equities)

**Investor Profiles**
1. **Aggressive** (high risk tolerance)
2. **Moderate** (medium risk tolerance)
3. **Conservative** (low risk tolerance)
Relationship with Portfolio Theory

The asset allocation puzzle

Mean–variance

Expected Return (in %)

Risk (in %)

Equity

Sharpe

Bond

Cash

Optimal Portfolio

Expected Return (in %)

Risk (in %)

(A)

(M)

(C)

Diversified Funds

Expected Return (in %)

Risk (in %)

(Dyn.)

(Def.)

Target Date Funds

Expected Return (in %)

Risk (in %)

20Y

30Y

40Y

60Y

50Y

We consider a backtest with MSCI World (hedged in EUR) and EuroMTS 10Y-15Y.

- Deleverage of an equity exposure
- Diversification in weights ≠ Risk diversification
- No mapping between fund profiles and investor profiles
Some Partial Answers

1. Cash is a risky asset in the long term.
2. Bonds have not the same maturity.

See Campbell and Viciera (2002), Bajeux-Besnainou et al. (2003), Cocco et al. (2005) or Munk and Sørensen (2010).
We consider a universe of $n$ assets. We denote by $\mu$ the vector of their expected returns and by $\Sigma$ the corresponding covariance matrix. We specify the optimization problem as follows:

$$\min \frac{1}{2} w^\top \Sigma w$$

subject to:

$$\begin{align*}
1^\top w &= 1 \\
\mu^\top w &\geq \mu^* \\
w &\in \mathbb{R}^n \cap \mathcal{C}
\end{align*}$$

where $w$ is the vector of weights in the portfolio and $\mathcal{C}$ is the set of weights constraints. We define:

- the unconstrained portfolio $w^*$ or $w^*(\mu, \Sigma)$:
  $$\mathcal{C} = \mathbb{R}^n$$

- the constrained portfolio $\tilde{w}$:
  $$\mathcal{C}(w^-, w^+) = \{ w \in \mathbb{R}^n : w_i^- \leq w_i \leq w_i^+ \}$$
Theorem

Jagannathan and Ma (2003) show that the constrained portfolio is the solution of the unconstrained problem:

\[ \tilde{w} = w^* (\tilde{\mu}, \tilde{\Sigma}) \]

with:

\[
\begin{cases}
\tilde{\mu} = \mu \\
\tilde{\Sigma} = \Sigma + (\lambda^+ - \lambda^-) \mathbf{1}^T + \mathbf{1} (\lambda^+ - \lambda^-)^T
\end{cases}
\]

where \( \lambda^- \) and \( \lambda^+ \) are the Lagrange coefficients vectors associated to the lower and upper bounds.
Main Result

Interpretation

We have \( \tilde{\Sigma}_{i,j} = \Sigma_{i,j} + \Delta_{i,j} \) with:

\[
\begin{array}{ccc}
(\Delta)_{i,j} & w_i^- & w_i^-, w_i^+ \\
w_j^- & -\left(\lambda_i^- + \lambda_j^-\right) & -\lambda_j^- & \lambda_i^+ - \lambda_j^- \\
w_j^- \quad , \quad w_j^+ & -\lambda_i^- & 0 & \lambda_i^+ \\
w_j^+ & \lambda_j^+ - \lambda_i^- & \lambda_j^+ & \lambda_i^+ + \lambda_j^+
\end{array}
\]

The perturbation \( \Delta_{i,j} \) may be negative, nul or positive.

1. For the volatility, we obtain \( \tilde{\sigma}_i = \sqrt{\sigma_i^2 + \Delta_{i,i}} \).

\[
\begin{array}{ccc}
w_i^- & w_i^-, w_i^+ & w_i^+ \\
\tilde{\sigma}_i < \sigma_i & \tilde{\sigma}_i = \sigma_i & \tilde{\sigma}_i > \sigma_i
\end{array}
\]

2. For the correlation, we obtain \( \tilde{\rho}_{i,j} = \frac{\rho_{i,j} \sigma_i \sigma_j + \Delta_{i,j}}{\sqrt{(\sigma_i^2 + \Delta_{i,i})(\sigma_j^2 + \Delta_{j,j})}} \).

\( \Rightarrow \) Similar to the Black-Litterman approach.
Proof for the Global Minimum Variance Portfolio

We define the Lagrange function as
\[
 f(w; \lambda_0) = \frac{1}{2} w^\top \Sigma w - \lambda_0 (1^\top w - 1)
\]
with \( \lambda_0 \geq 0 \). The first order conditions are \( \Sigma w - \lambda_0 1 = 0 \) and \( 1^\top w - 1 = 0 \). We deduce that the optimal solution is:
\[
 w^* = \lambda^*_0 \Sigma^{-1} 1 = \frac{1}{1^\top \Sigma 1} \Sigma^{-1} 1
\]

With weights constraints \( C(w^-, w^+) \), we have:
\[
 f(w; \lambda_0, \lambda^-, \lambda^+) = \frac{1}{2} w^\top \Sigma w - \lambda_0 (1^\top w - 1) - \lambda^-^\top (w - w^-) - \lambda^+^\top (w^+ - w)
\]
with \( \lambda_0 \geq 0 \), \( \lambda^- \geq 0 \) and \( \lambda^+ \geq 0 \). In this case, the first-order conditions becomes \( \Sigma w - \lambda_0 1 - \lambda^- + \lambda^+ = 0 \) and \( 1^\top w - 1 = 0 \). We have:
\[
 \tilde{\Sigma} \tilde{w} = \left( \Sigma + (\lambda^+ - \lambda^-) 1^\top + 1 (\lambda^+ - \lambda^-)^\top \right) \tilde{w} = \left( 2\tilde{\lambda}_0 - \tilde{w}^\top \Sigma \tilde{w} \right) 1
\]

Because \( \tilde{\Sigma} \tilde{w} \) is a constant vector, it proves that \( \tilde{w} \) is the solution of the unconstrained optimisation problem with \( \lambda^*_0 = \left( 2\tilde{\lambda}_0 - \tilde{w}^\top \Sigma \tilde{w} \right) \).
Table: Specification of the covariance matrix $\Sigma$ (in %)

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<tr>
<th>$\sigma_i$</th>
<th>$\rho_{i,j}$</th>
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<td>100.00</td>
</tr>
<tr>
<td>20.00</td>
<td>10.00 100.00</td>
</tr>
<tr>
<td>25.00</td>
<td>40.00 70.00 100.00</td>
</tr>
<tr>
<td>30.00</td>
<td>50.00 40.00 80.00 100.00</td>
</tr>
</tbody>
</table>

Given these parameters, the **global minimum variance portfolio** is equal to:

$$w^* = \begin{pmatrix} 72.742\% \\ 49.464\% \\ -20.454\% \\ -1.753\% \end{pmatrix}$$
Table: Global minimum variance portfolio when $w_i \geq 10\%$

<table>
<thead>
<tr>
<th>$\tilde{w}_i$</th>
<th>$\lambda_i^-$</th>
<th>$\lambda_i^+$</th>
<th>$\tilde{\sigma}_i$</th>
<th>$\tilde{\rho}_{i,j}$</th>
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<tr>
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<td>1.625</td>
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<td>23.980</td>
<td>17.378</td>
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Table: Global minimum variance portfolio when $0\% \leq w_i \leq 50\%$

<table>
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<th>$\tilde{w}_i$</th>
<th>$\lambda_i^-$</th>
<th>$\lambda_i^+$</th>
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<td>0.000</td>
<td>30.000</td>
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**Table:** MSR portfolio when $0\% \leq w_i \leq 40\%$ and $\mathbf{sh}^* = 0.5$

<table>
<thead>
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<th>$\tilde{w}_i$</th>
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We obtain:

$$\tilde{\mathbf{sh}} = \begin{pmatrix} 0.381 \\ 0.444 \\ 0.5 \\ 0.5 \end{pmatrix}$$
Backtest with monthly rebalancing and one-year empirical covariance matrix.

Lower bound is set to 0% and upper bound is set to 5%.

We define:

- the mean absolute deviations as $\delta_\sigma = \frac{1}{n} \sum_{i=1}^{n} |\tilde{\sigma}_{i,t} - \sigma_{i,t}|$ for the volatility and $\delta_\rho = \frac{2}{n(n-1)} \sum_{i>j} |\tilde{\rho}_{i,j,t} - \rho_{i,j,t}|$ for the correlation;
- the maximum of absolute deviations as $\delta_\sigma^+ = \max_i |\tilde{\sigma}_{i,t} - \sigma_{i,t}|$ for the volatility and $\delta_\rho^+ = \max_{i,j} |\tilde{\rho}_{i,j,t} - \rho_{i,j,t}|$ for the correlation;
- The peak-over-threshold frequency: $\pi_{\rho}(x) = \frac{2}{n(n-1)} \sum_{i>j} 1 \{ |\tilde{\rho}_{i,j,t} - \rho_{i,j,t}| > x \}$. 
Application to the DJ Eurostoxx 50
Impact (in %) on the volatilities for the MIN portfolio

- Mean of absolute deviations $\delta_x$
- Maximum of absolute deviations $\delta_+^x$
- Kendall $\tau$ statistic

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Quantitative Methods in Asset Management
Application to the DJ Eurostoxx 50
Impact (in %) on the correlations for the MIN portfolio

Statistic $\delta_\rho$

Statistic $\delta_\rho^+$

Statistic $\pi_\rho(5\%)$

Statistic $\pi_\rho(10\%)$
Application to the DJ Eurostoxx 50

Impact (in %) on the risk factors for the MIN portfolio

1st risk factor

2nd risk factor

3rd risk factor

4th-10th risk factors
Application to the DJ Eurostoxx 50
Density of the implied Sharpe ratio for the MSR portfolio

January 2004

January 2006

January 2008

January 2009

Thierry Roncalli
Quantitative Methods in Asset Management
Option Trading & Asset Management strategies

Option Trading strategy \(\ne\) Asset Management strategy

- Arbitrage theory
- Asset prices probability distribution (risk-neutral / historical)
- Management style (formula-based / systematic / discretionary)
- Mathematical tools (Itô calculus / statistics)

A long-position in a call option is equivalent to a long position in the underlying:

\[ e_t = \Delta_t \]

⇒ A call option = trend-following (long-only) strategy.
⇒ The cost of a call option = call premium.
We consider a systematic strategy where the number \( n_t \) of invested shares depends on the price asset \( S_t \): \( n_t = f(S_t) \). Let \( X_t \) be the value of the strategy (or the fund). The risky exposure and the dynamic of the fund are given by:

\[
\begin{align*}
e_t &= n_t \frac{S_t}{X_t} = f(S_t) \frac{S_t}{X_t} \\
dX_t &= f(S_t) \, dS_t
\end{align*}
\]

If we assume that \( dS_t = \mu(S_t) \, dS_t + \sigma_t S_t \, dS_t \), Bruder and Gaussel (2010) show that:

\[
X_T = X_0 + \int_{S_0}^{S_T} f(S) \, dS - \frac{1}{2} \int_0^T \partial_S f(S_t) S_t^2 \sigma_t^2 \, dt
\]

Example: Stop loss, Take profit, etc.
Application to a Trend-following Strategy

1. **Long only** \( f(S_t) = mS_t \):
   \[
   F(S_T) = S_0 + \frac{1}{2} m (S_T^2 - S_0^2)
   \]

2. **Long short** \( f(S_t) = m(S_t - S^*) \):
   \[
   F(S_T) = S_0 + \frac{1}{2} m \left( (S_T - S^*)^2 - (S_0 - S^*)^2 \right)
   \]

For these two cases, the cost function is:
\[
C_T = \frac{1}{2} m \int_0^T S_t^2 \sigma_t^2 dt \geq 0
\]

This simple model explained some stylized facts of the CTA strategy (leverage, volatility, trends, long-term / short-term, etc.)
Some Issues

Market Timing
- Behavioral (speculation) model
- No risk premium

Tactical Asset Allocation
- Business cycle
- Time-varying risk premium


Strategic Asset Allocation
- Growth model
- Stationary risk premium

Solow (1956), fundamental approach.

MT
1 Day - 1 Month

TAA
3 Months - 3 Years

SAA
10 Years - 50 Years
Our Approach

1. A comprehensive framework
2. Distinction between TAA and SAA
3. Based on economic models (Solow model, Golden rule, Philipps curve, Okun’s law, NAIRU, etc.)
4. Understanding the concept of risk premium (and its link to the cointegration theory)
5. Sensitivity and Scenario analysis
Economic Modeling of Asset Returns

Economic Pillars

- Economic growth
  1. Solow model
  2. Golden rule

- Monetary policy and inflation
  1. Phillips curve
  2. NAIRU
Economic Modeling of Asset Returns
Asset Return and Risk Premium

The Two Economic Pillars

Potential Growth  Inflation

⇒

Long-run Returns on Asset Classes

Short Rate  ➞  Government Bonds  ➞

Equities
Corporate Bonds
Commodities
Other Asset Classes
We propose to derive long-run short rates $r_{\infty}$ from the lower bound of the normative Golden rule:

$$r_{\infty} = g_{\infty} + \pi_{\infty}$$

where $g_{\infty}$ is the long-run real potential output growth and $\pi_{\infty}$ is the long-run inflation.

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<td>9.1%</td>
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</tbody>
</table>
Economic Modeling of Asset Returns

Sovereign Bonds

The long-run value of the nominal bond yield $R^b$ is equal to:

$$R^b = R^b_{\infty} + \pi_{\infty}$$

where $R^b_{\infty}$ is the long-run real bond yield $R^b$ and $\pi_{\infty}$ is the long-run inflation.

To estimate $R^b_{\infty}$, we consider the following regression model:

$$R^b_t = \beta_0 + \beta_1 r_t + \beta_2 \sigma^{\pi}_t + \beta_3 (B/Y)_t + \epsilon_t$$

where $r_t$ is the real short rate, $\sigma^{\pi}_t$ is the inflation risk and $(B/Y)_t$ is the government balance on output ratio (proxy for debt risk).
Economic Modeling of Asset Returns

Risky Bonds

The long-run bond yield $R^{cr}_\infty$ is equal to:

$$R^{cr}_\infty = R^b_\infty + s^{cr}_\infty$$

where $R^b_\infty$ is the US long-run bond yield and $s^{cr}_\infty$ is the long-run spread.

For the investment grade and high yield spreads, the regression model is:

$$s^{cr}_t = \beta_0 + \beta_1 \sigma^e_t + \beta_2 g_t + \varepsilon_t$$

where $\sigma^e_t$ denotes the equity volatility and $g_t$ is the output growth. For the emerging bond spread, the regression model becomes:

$$s^{cr}_t = \beta_0 + \beta_1 \sigma^e_t + \beta_2 (CA/Y)_t + \varepsilon_t$$

where $(CA/Y)_t$ is the current account on output ratio.
Economic Modeling of Asset Returns

Table: Economic forecast of the 10-year bond yield

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<thead>
<tr>
<th></th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2050</th>
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<td>IG EURO</td>
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Economic Modeling of Asset Returns

Expected returns of bonds are deduced from the economic forecast of the 10-year bond yield using a sensitivity/duration hypothesis.

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</tbody>
</table>
Economic Modeling of Asset Returns

Equities

The long-run equity return is equal to:

\[ R^e_e = R^b_e + \bar{R}^e_e \]

where \( R^b_e \) is the long-run bond yield and \( \bar{R}^e_e \) is the equity excess return.

The regression model is:

\[ \bar{R}^e_{t+10} = \beta_0 + \beta_1 PE_t + \beta_2 R^b_t + \varepsilon_t \]

where \( PE_t \) is the price earning ratio and \( R^b_t \) is the 10-year bond yield.

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<td>-1.2%</td>
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<td>16.8%</td>
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<td>8.2%</td>
<td>8.7%</td>
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<tr>
<td>JAPAN</td>
<td>20.1%</td>
<td>-0.5%</td>
<td>-3.4%</td>
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<td>10.7%</td>
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</table>
Economic Modeling of Asset Returns

Other assets

1. Small cap
2. Commodities
3. Hedge funds
4. Real estate
5. Foreign exchanges

- Liquidity risk
- Globalization / Convergence
- Ressources / Consumption
Economic Modeling of Volatility and Correlation

Volatility
- Historical figures
- Mean-reverting properties
- Macro-economic volatility
- Tail risks

Correlation
- Historical figures
- Time-varying correlations
- **Flight-to-quality** & globalization
- Inflation regime $\Rightarrow$ bond-stock correlation
Strategic Asset Allocation in Practice

- **US** \((\sim)\)
- **EURO** \((-)\)
- **JAPAN** \((-\sim)\)
- **PACIFIC** \((+)\)
- **EM** \((+++)\)

![Graph showing expected return vs. volatility](image-url)
Strategic Asset Allocation in Practice

Bond-Equity Allocation Policy

Figure: Average allocation of European pension funds
## Strategic Asset Allocation

### Bond-Equity Allocation Policy

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<th>Weights Bond</th>
<th>Weights Equity</th>
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</table>

**Strong diversification effect.**

Standard risk-aversion for long-term investors: $\gamma = 5$. 
Alternative assets = substitute of equities (not of bonds).

The $2/3 - 1/3$ rule (for risk-seeking long-term investors).

Liquidity risk $\implies$ Tactical asset allocation.
Sensitivity and Scenario Analysis

Economic scenario
- Expectation → Probability
- Stress scenario

Risk premium
- Confidence intervals
- Scenario analysis

Table: Coefficient estimates for bond regressions

<table>
<thead>
<tr>
<th>Study Period</th>
<th>Constant</th>
<th>$\tau_t$</th>
<th>$\sigma^\tau_t$</th>
<th>$(B/Y)_t$</th>
<th>$R^2$</th>
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<tr>
<td>US 1982–2009</td>
<td>0.008 (2.006)</td>
<td>0.59 (3.63)</td>
<td>0.67 (1.51)</td>
<td>$-0.11 (-1.00)$</td>
<td>0.82</td>
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<tr>
<td>EURO 1982–2009</td>
<td>0.007 (1.988)</td>
<td>0.47 (4.15)</td>
<td>2.03 (2.07)</td>
<td>$-0.10 (-0.84)$</td>
<td>0.94</td>
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<tr>
<td>JAPAN 1982–2009</td>
<td>0.011 (3.379)</td>
<td>0.66 (5.57)</td>
<td>0.21 (1.82)</td>
<td>$-0.05 (-1.10)$</td>
<td>0.85</td>
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<tr>
<td>PACIFIC 1982–2009</td>
<td>0.017 (1.212)</td>
<td>0.47 (3.96)</td>
<td>0.15 (0.19)</td>
<td>$-0.32 (-2.42)$</td>
<td>0.69</td>
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</table>
Asset Management $\implies$ some complex problems:

- Benchmarking
- Portfolio allocation
- Long-term risks
- SAA vs TAA
- Momentum strategies
- Statistical arbitrage
- etc.

Quantitative methods = a tool to understand (and sometimes to solve) these problems.
Dynamic Asset Allocation for Stocks, Bonds, and Cash.

J.Y. Campbell, L.M. Viciera.
Strategic Asset Allocation.
*Oxford University Press, 2002.*

J.F. Cocco, F.J. Gomes, P.J. Maenhout.
Consumption and Portfolio Choice over the Life Cycle.

S. Darolles, K. Eychenne, S. Martinetti.
For Further Reading II

For Further Reading III