Impatience and myopia through belief functions

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Motivation

$B_\infty(\mathbb{N})$ be the set of bounded real valued functions defined on $\mathbb{N}$

Interpret $x \in B_\infty(\mathbb{N})$ as a countable income (consumption) stream

Let $\succsim$ be a weak order on $B_\infty(\mathbb{N})$

Rank the $x$’s through the Choquet integral of $x$ w.r.t. a capacity $\nu$ i.e. through $\int x \, d\nu$

$\nu : \mathcal{P}(\mathbb{N}) \to [0, 1]$ s.t. $\nu(\emptyset) = 0, \nu(\mathbb{N}) = 1$, and $A \subseteq B \Rightarrow \nu(A) \leq \nu(B)$.

By definition

$$\int x \, d\nu = \int_{-\infty}^{0} \left(\nu(x \geq t) - 1\right) dt + \int_{0}^{+\infty} \nu(x \geq t) \, dt.$$
Justification for the use of $\int x \, d\nu$:

**Comonotonicity**

Two income streams $x$ and $y$ are comonotonic if for all $(s, t) \in \mathbb{N}^2$,

$$(x_s - x_t)(y_s - y_t) \geq 0.$$ 

**Comonotonic independence**

$x, y, z \in B_\infty(\mathbb{N})$, if $z$ is comonotonic with $x$ and $y$, and $x \sim y$ then $x + z \sim y + z$. 
More precisely

Specifying more precisely \( \nu \) in multi-period decisions:


Sequential comonotonicity

Two income streams \( x \) and \( y \) are sequentially comonotonic (s.c.) if for all \( n \in \mathbb{N} \), \((x_{n+1} - x_n)(y_{n+1} - y_n) \geq 0\).

Axiom of variation aversion

\( x, y, z \in B_\infty(\mathbb{N}) \) if \( \{y, z\} \) are sequentially comonotonic and \( x \sim y \), then \( x + z \succeq y + z \).

Smoothing two successive incomes can be considered as an improvement by the Decision Maker.
Variation aversion implies **convexity** of preferences

\[ x \sim y \Rightarrow \alpha x + (1 - \alpha)y \succ y, \quad \forall \alpha \in (0, 1). \]

hence \( \nu \) is convex (monotone of order 2) i.e.

\[ \nu(A \cup B) + \nu(A \cap B) \geq \nu(A) + \nu(B) \text{ for all } A, B \in \mathcal{P}(\mathbb{N}). \]

Actually, variation aversion implies monotonicity of infinite order i.e. \( \nu \) is a belief function.
Belief functions

A belief function \( \nu \) on \( \mathcal{P}(\mathbb{N}) \) is a belief function if \( \nu \) is a capacity and if \( \nu \) is \( \infty \)-monotone, i.e., for every \( k \geq 2 \), \( B_1, \ldots, B_k \in \mathcal{B} = \mathcal{P}(\mathbb{N}) \),

\[
\nu\left(\bigcup_{j=1}^{k} B_j\right) \geq \sum_{J, \emptyset \neq J \subseteq \{1, \ldots, k\}} (-1)^{|J|+1} \nu\left(\bigcap_{j \in J} B_j\right)
\]

We now assume \( \nu \) being a belief function.
Motivation

Impatience

\[ \succsim \text{ is impatient if} \]

\[ \forall x \in B_\infty(\mathbb{N}), \forall \epsilon > 0, \exists N_o(x, \epsilon) \in \mathbb{N}, \left[ n \geq N_o \Rightarrow (x + \epsilon)1_{[0,n]} \succ x \right]. \]

Proposition [Chateauneuf and Rébillé (2004)]

(i) \( \succsim \) is impatient

\( \iff \)

(ii) \( \nu \) is continuous at \( \mathbb{N} \) i.e. \( B_n \uparrow \mathbb{N} \Rightarrow \nu(B_n) \uparrow 1 \)

\( \iff \)

(iii) [Rosenmüller (1972)] \( \nu \) is continuous i.e. \( \forall B_n, B \in \mathcal{B}, B_n \uparrow B \Rightarrow \nu(B_n) \uparrow \nu(B), \text{ and } B_n \downarrow B \Rightarrow \nu(B_n) \downarrow \nu(B) \)
Motivation

Myopia

Myopia

\( \succcurlyeq \) is myopic if

\[ \forall x, y \in B_\infty(\mathbb{N}) \text{ s.t. } x \succ y, \forall \epsilon > 0, \exists N_o \in \mathbb{N}, \left[ n \geq N_o \Rightarrow x \succ y + \epsilon \mathbf{1}_{[n,+\infty)} \right]. \]

Proposition [Chateauneuf and Ventura (2010)]

(i) \( \succcurlyeq \) is myopic

\( \iff \)

(ii) \( \nu \) is outer-continuous i.e. \( \forall B_n, B \in \mathcal{B}, B_n \downarrow B \Rightarrow \nu(B_n) \downarrow \nu(B) \)
1. How to build continuous or outer-continuous belief functions on $(\mathbb{N}, \mathcal{P}(\mathbb{N}))$.

2. More generally, how to build belief functions on polish spaces $(\Omega, \mathcal{B}(\Omega))$ with specific continuity properties.

3. How are impatience and myopia translated in continuous time i.e. if $\mathbb{N}$ is replaced by $\Omega = [0, +\infty)$ and $\mathcal{B}_\infty(\mathbb{N})$ replaced by $L^\infty$ the space of bounded real-valued measurable functions on $(\Omega, \mathcal{B})$. 
Let $\Omega$ be finite, $\mathcal{B} = 2^\Omega$, and $\mathcal{B}' = \mathcal{B}\setminus\{\emptyset\} = 2^\Omega\setminus\{\emptyset\}$.

For $A \in \mathcal{B}'$, the **unanimity game** $u_A : \mathcal{B} \to [0, 1]$ is defined by

$$u_A(B) = \begin{cases} 1 & \text{if } A \subset B, \\ 0 & \text{otherwise}. \end{cases}$$

Any unanimity game is a belief function.

Unanimity games are the extremal elements of the compact convex set of belief functions on $(\Omega, 2^\Omega)$. 
The finite case

Proposition 1

\( \nu \) is a belief function on \((\Omega, 2^\Omega)\) if and only if there exists a probability measure \( \mu_\nu \) on \((\mathcal{B}', 2^{\mathcal{B}'})\) such that

\[
\nu = \sum_{A \in \mathcal{B}'} \mu_\nu(A) u_A
\]

Furthermore \( \mu_\nu \) is unique and called the Möbius inverse of \( \nu \).

Apply Krein-Milman Theorem
[Choquet (1954)]’s theorem

Let $K$ be a nonempty compact convex subset of a locally convex and Hausdorff topological vector space $E$.

Denote by $A(K)$ the space of affine continuous functions on $K$.

A function $f : K \rightarrow \mathbb{R}$ is said to be affine if

$$f(tx + (1-t)y) = tf(x) + (1-t)f(y) \quad \forall x, y \in K, 0 \leq t \leq 1.$$ 

A point $x \in K$ is called extreme point of $K$ if from $y, z \in K$ and $ty + (1-t)z = x$ with $0 < t < 1$ we get $x = y = z$.

Denote $\text{ext}K$ the set of extreme points of $K$. 
The finite case

Choquet’s theorem (1954)

Theorem 1

For every \( x \in K \), there exists a probability measure (\( \sigma \)-additive) \( \mu_x \) on \( \text{ext} K \) (w.r.t. the smallest \( \sigma \)-algebra making all elements of \( A(K)|_{\text{ext} K} \) measurable) such that for all \( f \in A(K) \):

\[
f(x) = \int_{\text{ext} K} f|_{\text{ext} K} \, d\mu_x.
\]
Characterization of continuity properties of belief functions $\nu$ defined on a Polish space $\Omega$, through specific related $\sigma$-additive Möbius transform $\mu_{\nu}$:

Let $\mathcal{V}$ be the set of all games defined on $\mathcal{B}$ i.e.

$$\mathcal{V} = \{\nu : \mathcal{B} \rightarrow \mathbb{R}, \nu(\emptyset) = 0\}.$$ 

Let $E$ be a linear subspace of $\mathcal{V}$. Endow $E$ with the topology $\tau$ of simple convergence, as in [Marinacci (1996)].

$(E, \tau)$ is a locally convex and Hausdorff topological space.

The set $K = \text{Bel}_E$ of belief functions in $E$ is $\tau$-compact and convex.
For every $B \in \mathcal{B}$ the mapping

$$f : \nu \in K \mapsto f(\nu) = \nu(B)$$

is affine and continuous, hence $f \in A(K)$.

So, as soon as belief functions with given continuity properties are considered and the related space of games is a linear subspace of $\mathcal{V}$, then one can apply Choquet’s theorem, once extremal elements of $K = Bel_E$ are characterized.
When the set $\Omega$ is finite, Choquet's theorem gives:

For every belief function $\nu$, there exists a probability measure $\mu_{\nu}$ on $(\mathcal{B}', 2^{\mathcal{B}'})$ such that, for every $B \in \mathcal{B}$:

$$\nu(B) = \int_{\text{ext Bel}_E} u(B) d\mu_{\nu}(u) = \sum_{A \in \mathcal{B}'} u_A(B) \mu_{\nu}(A)$$

Or else $\nu(B) = \mu_{\nu}(\tilde{B})$ where $\tilde{B} = \{ A \in \mathcal{B}' : A \subset B \}$

or equally $\nu(B) = \sum_{A \subseteq B} \mu_{\nu}(A)$ (writing $\mu_{\nu}(A)$ instead of $\mu_{\nu}([A])$).
A useful known result (Choquet (1954))

The extreme points of the set of belief functions defined on a measurable space $(\Omega, \mathcal{B})$ are the filter games

A nonempty set $p$ of elements of $\mathcal{B}$ is called a filter if

1. $\forall A, B \in \mathcal{B}, [A \in p, A \subset B \Rightarrow B \in p]$,  
2. $\forall A, B \in \mathcal{B}, [A, B \in p \Rightarrow A \cap B \in p]$.

The filter $p$ is called proper if $\emptyset \notin p$.

A game $\nu : \mathcal{B} \rightarrow \{0, 1\}$ is called a filter game if the set $p := \{ B \in \mathcal{B} : \nu(B) = 1 \}$ is a filter. In this case $\nu$ is denoted $u_p$ where $u_p$ is obviously defined by $u_p(B) = 1$ if $B \in p$, and $u_p(B) = 0$ otherwise.
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A belief function $\nu$ is outer-continuous at $B \in \mathcal{B}$ if

$$B_n \in \mathcal{B}, B_n \downarrow B \implies \nu(B_n) \downarrow \nu(B)$$

A game $\nu \in \mathcal{V}$ is outer-continuous if it is outer-continuous at every $B \in \mathcal{B}$ i.e. if $\forall B \in \mathcal{B}, B_n \in \mathcal{B}, B_n \downarrow B$ implies $\nu(B_n) \xrightarrow[n \to \infty]{} \nu(B)$
Outer-continuous belief functions

$E_o$ denotes the linear space of bounded, outer-continuous games

$Bel_{E_o}$ denotes the compact convex set of outer-continuous belief functions

$\text{ext } Bel_{E_o}$ is the set of extreme elements of $Bel_{E_o}$.

Proposition 2

$\text{ext } Bel_{E_o} = \{u_p : p \text{ is a proper filter closed under countable intersection}\}$. 
Denote by $\Sigma_o$ the $\sigma$-algebra on $\text{ext } \text{Bel}_{E_0}$ generated by the family $\{ \tilde{B} : B \in \mathcal{B}, B \neq \emptyset \}$, where

$$\tilde{B} = \{ u_p \in \text{ext } \text{Bel}_{E_0} : B \in p \}.$$

**Theorem 2**

For every $\nu \in \text{Bel}_{E_0}$ there exists a $\sigma$-additive measure $\mu_\nu$ on $\Sigma_o$ such that for all $B \in \mathcal{B}$,

$$\nu(B) = \int_{\text{ext } \text{Bel}_{E_0}} u(B) \, d\mu_\nu(u) = \mu_\nu(\tilde{B}). \quad (1)$$

Conversely, given a $\sigma$-additive measure $\mu_\nu$ on $\Sigma_o$, the expression above defines an outer-continuous belief function on $\mathcal{B}$. 

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The particular case of \( \Omega \) countable (i.e. Myopia when \( \Omega = \mathbb{N} \)):

**Lemma 1**

*Every proper filter \( p \) on \( \mathbb{N} (p \neq 2^\mathbb{N}) \) closed under countable intersection is principal, that is, there exists a nonempty subset \( T \) of \( \mathbb{N} \) such that \( p = \{ B \subseteq \mathbb{N} : T \subseteq B \} \).*
Countable state space

Since a countable set endowed with the discrete topology is a polish space, we obtain as a corollary:

**Corollary 1**

*(Theorem D. [Gilboa and Schmeidler (1995)])*

If $\Omega$ is countable, for every $\nu \in \text{Bel}_{E_o}$ there exists a $\sigma$-additive measure $\mu_\nu$ on the $\sigma$-algebra generated by the sets $\tilde{B} := \{ T \in 2^\Omega : \emptyset \neq T \subseteq B \}$ for $B \subset \Omega$, $B \neq \emptyset$, such that

$$\nu(B) = \mu_\nu(\tilde{B}) \quad \text{for all} \quad B \in 2^\Omega.$$  

Conversely, given a $\sigma$-additive measure $\mu_\nu$ on the $\sigma$-algebra generated by the sets $\tilde{B}$, the expression above defines an outer-continuous belief function on $2^\Omega$. 
The following corollary provides a simple way to build an outer-continuous belief function on $2^\Omega$ when $\Omega$ is countable.

**Corollary 2**

Let $C$ be a countable subset of $2^\mathbb{N} \setminus \{\emptyset\}$ and let $m : 2^\mathbb{N} \rightarrow [0, 1]$ be such that

$$\sum_{\{T : T \in C\}} m(T) = 1 \text{ and } m|_{C^c} = 0.$$  

Define a game $\nu$ by:

$$\nu(B) = \sum_{\{T : T \in C, T \subset B\}} m(T) \text{ for all } B \in 2^\mathbb{N}.$$  

Then $\nu$ is an outer-continuous belief function on $2^\mathbb{N}$.
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A belief function $\nu$ is $G$-inner-continuous at $G \in \mathcal{G} := \{\text{open sets}\}$ if 

$$G_n \in \mathcal{G}, \; G_n \uparrow G \Rightarrow \nu(G_n) \uparrow \nu(G)$$

A game $\nu \in \mathcal{V}$ is $G$-inner-continuous if it is $G$-inner-continuous at every $G \in \mathcal{G}$ i.e. if $\forall G \in \mathcal{G}, \; G_n \in \mathcal{G}, \; G_n \uparrow G$ implies $\nu(G_n) \xrightarrow{n \to \infty} \nu(G)$
$E_{o,G}$ denotes the linear space of bounded, outer-continuous and $G$-inner-continuous games.

$Bel_{E_{o,G}}$ denotes the compact convex set of outer-continuous and $G$-inner-continuous belief functions.

$\text{ext } Bel_{E_{o,G}}$ is the set of extreme elements of $Bel_{E_{o,G}}$.

$\mathcal{K} = \{\text{compact subsets } K \text{ of } \Omega\}$

**Proposition 3**

$$\text{ext } Bel_{E_{o,G}} = \{ u_K : K \neq \emptyset, K \in \mathcal{K} \}.$$
The proof of Proposition 3 relies on the following capacitability theorem.

**Capacitability theorem** [Choquet (1954)], [Debs et al (1999)]

Let $\nu$ be a convex cocapacity on a polish space $(\Omega, \mathcal{B})$ i.e. $\nu$ is $\mathcal{G}$-inner-continuous and outer-continuous then

$$\nu(B) = \sup_{K \in \mathcal{K}, K \subseteq B} \nu(K) = \inf_{G \in \mathcal{G}, G \supseteq B} \nu(G) \quad \forall B \in \mathcal{B}.$$
Denote by $\Sigma_{o,G}$ the $\sigma$-algebra on $\text{ext}\, Bel_{E_0,G}$ generated by the family
\{ $\tilde{B} : B \in \mathcal{B}, B \neq \emptyset$\}, where
$$\tilde{B} = \{ u_K : K \in \mathcal{K}, \emptyset \neq K \subseteq B \}.$$ 

### Theorem 3

For every $\nu \in Bel_{E_0,G}$ there exists a $\sigma$-additive measure $\mu_{\nu}$ on $\Sigma_{o,G}$ such that for all $B \in \mathcal{B}$,

$$\nu(B) = \int_{\text{ext}\, Bel_{E_0,G}} u(B) \, d\mu_{\nu}(u) = \mu_{\nu}\left( \{ u_K : K \in \mathcal{K}, \emptyset \neq K \subseteq B \} \right). \quad (2)$$

Conversely, given a $\sigma$-additive measure $\mu_{\nu}$ on $\Sigma_{o,G}$, the expression above defines an outer-continuous and $\mathcal{G}$-inner-continuous belief function on $\mathcal{B}$.
Indeed, Theorem 3 would allow to build simple outer-continuous and $G$-inner-continuous belief functions by selecting a countable set of compacts and proceeding in the same way as in Corollary 2.
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Characterization of continuity properties of belief functions

(σ-continuity)

(i.e. Impatience when \( \Omega = \mathbb{N} \))

(i.e. Impatience when \( \Omega = \mathbb{N} \))

Reminder

A convex capacity \( \nu \) (hence a belief function) is \( \sigma \)-continuous if and only if \( \nu \) is continuous at \( \Omega \) i.e. \( B_n, B \in \mathcal{B}, B_n \uparrow \Omega \) implies \( \nu(B_n) \uparrow 1 \).

\( E_\sigma \) denotes the linear space of \( \sigma \)-continuous games i.e. of games \( \nu \) such that \( B_n \uparrow \Omega \Rightarrow \nu(B_n) \xrightarrow{n \to \infty} 1. \)

\( \text{Bel}_{E_\sigma} \) denotes the compact convex set of \( \sigma \)-continuous belief functions on \( (\Omega, \mathcal{B}) \)

\( \text{ext Bel}_{E_\sigma} \) is the set of extreme elements of \( \text{Bel}_{E_\sigma} \).

\( \mathcal{K}_0 = \{ \text{finite subsets } K \text{ of } \Omega \} \)
Proposition 4

\[ \text{ext } Bel_{E_\sigma} = \{ u_K : K \neq \emptyset, K \in \mathcal{K}_0 \} \].

Since for a belief function, \( \sigma \)-continuity implies outer-continuity and \( \mathcal{G} \)-inner-continuity, Proposition 4 relies on Proposition 3 where it is proved that

\[ \text{ext } Bel_{E_{o, \mathcal{G}}} = \{ u_K : K \neq \emptyset, K \in \mathcal{K} \} \].
Theorem 4

For every \( \nu \in \text{Bel}_{E_\sigma} \) there exists a \( \sigma \)-additive measure \( \mu_\nu \) on \( \Sigma_\sigma \) such that for all \( B \in \mathcal{B} \),

\[
\nu(B) = \int_{\text{ext Bel}_{E_\sigma}} u(B) \, d\mu_\nu(u) = \mu_\nu\left(\{u_K : K \in \mathcal{K}_0, \emptyset \neq K \subseteq B\}\right). \tag{3}
\]

Conversely, given a \( \sigma \)-additive measure \( \mu_\nu \) on \( \Sigma_\sigma \), the expression above defines an inner-continuous belief function on \( \mathcal{B} \).

Corollary 3

For every \( \nu \in \text{Bel}_{E_\sigma} \) and for every bounded measurable function \( f \) on \( \Omega \), we have

\[
\int f \, d\nu = \int_{\{u_K : K \neq \emptyset, K \in \mathcal{K}_0\}} \left[\min_K f\right] \, d\mu_\nu.
\]
Corollary 4

[Chateauneuf and Rébillé (2004)]

A game $\nu$ on $2^\mathbb{N}$ is an inner-continuous belief function if and only if there exists a unique game\(^a\) $m : 2^\mathbb{N} \rightarrow [0, 1]$ with $\sum_{K \in \mathcal{K}_0} m(K) = 1$ and $m|_{\mathcal{K}_0^c} = 0$ such that

$$\nu(B) = \sum_{\{K : K \in \mathcal{K}_0, K \subset B\}} m(K) \quad \forall B \in 2^\mathbb{N}. \quad (4)$$

Furthermore,

$$m(B) = \sum_{\{K : K \in \mathcal{K}_0, K \subset B\}} (-1)^{|B \setminus K|} \nu(K) \quad \forall B \in \mathcal{K}_0. \quad (5)$$

\(^a\)In particular $m(\emptyset) = 0$. 
Impatience and Myopia

$L^\infty(\Omega)$ the space of bounded real-valued measurable functions on $(\Omega, \mathcal{B})$ with $\mathcal{B}$ the borelians of $\Omega$

Interpret $x \in L^\infty(\Omega)$ as a continuous income stream

\(\succeq\) a weak order on $L^\infty(\Omega)$ representable by the Choquet integral w.r.t. a belief function $\nu$ on $\Omega$
Impatience and Myopia in continuous time ($\Omega = \mathbb{R}_+$)

Impatience and Myopia

Myopia in continuous time

$\succsim$ is myopic if for every $(B_n)_n \subset \mathcal{B}$ such that $B_n \downarrow \emptyset$, for every $x, y$ in $L^\infty$, and $c$ in $\mathbb{R}$: $x \succ y$ implies $x \succ y + c \mathbf{1}_{B_n}$ for sufficiently large $n$.

Theorem 5

$\succsim$ is myopic if and only if $\nu$ is outer-continuous.
Impatience in continuous time

\( \succcurlyeq \) is impatient if

\[ \forall x \in L_\infty, \forall \epsilon > 0, \exists T_0(x, \epsilon) \in \mathbb{R}_+, \left[ T \geq T_0 \Rightarrow (x + \epsilon)1_{[0,T]} \succ x \right]. \]

Theorem 6

\( \nu \) is \( \mathcal{G} \)-inner-continuous \( \Rightarrow \) \( \succcurlyeq \) is impatient.
Strong Impatience

Notation: Let \((I_n)_{n \in \mathbb{N}}\) be disjoint open intervals of \(\mathbb{R}_+\) where \(I_1 = [0, b_1)\), \(I_2 = (a_2, +\infty)\), \(I_n = (a_n, b_n)\) otherwise.

Denote by \(J_n = [c_n, d_n]\) closed intervals strictly included in \(I_n\) namely \(a_n < c_n < d_n < b_n\) for all \(n \in \mathbb{N}\).

Theorem 7

\(\succsim\) is strongly impatient if and only if \(\nu\) is \(G\)-inner-continuous.
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