Modeling urban housing market dynamics: can the socio-spatial segregation preserve some social diversity?

Laetitia Gauvin\(^1,\)*, Annick Vignes\(^2\), Jean-Pierre Nadal\(^1,3\)

1. Laboratoire de Physique Statistique (LPS, UMR 8550 CNRS-ENS-UPMC-Univ. Paris Diderot), Ecole Normale Supérieure, Paris, France
2. Equipe de Recherche sur les Marchés, l’Emploi et la Simulation (ERMES, EA 4441 CNRS-Paris II), University Paris II Panthéon-Assas, Paris, France
3. Centre d’Analyse et de Mathématique Sociales (CAMS, UMR 8557 CNRS-EHESS), Ecole des Hautes Études en Sciences Sociales, Paris, France

* laetitia.gauvin@lps.ens.fr

“Don’t buy the house, buy the neighborhood” (Russian proverb). This paper is concerned with issues related to social segregation in urban environments. Going beyond the simplest models such as the one introduced by T. C. Schelling in the 70’s, we introduce a spatial model of real estate transactions between agents that are heterogeneous in their income and thus in their willingness to pay. The goal of the model is to see how the spatial income segregation depends on both economic constraints and social interactions. The housing market consists of assets differentiated by their location in the city and the agents choose according to these locations. A key feature of the model is the assumption that agents preferences for a location depend both on an intrinsic at-
tractiveness of the location, and on the social characteristics of its neighborhood. The demand for an asset thus depends on the local attractiveness. An hypothesis of the model is that the price of an asset also depends on the local attractiveness. Non-trivial buying/settling patterns emerge from the resulting dynamics. We first focus on the case of a monocentric city, i.e. with a highly attractive center. The stationary state of the market dynamics is analytically characterized and yields the distribution of income over space. We then show how these results extend to more complex non-monocentric cities. The model is also studied through numerical agent-based simulations. The joint analytical and numerical analysis reveal that, even if socio-spatial segregation occurs, some social diversity is preserved at most locations. The analytical resolution of the model highlights the existence of a critical endogeneous income threshold: agents with willingness to pay above this threshold can buy an asset wherever they demand. On the contrary, agents with a willingness to pay below the threshold can buy only in a restricted area. We then empirically verify the pertinence of these results through the analysis of a database of real-estate transactions in Paris. Some general trends are reproduced by the housing market model: the distribution of agents by income inside the city is characterized by a dissimilarity index that shows variations in the space comparable to those observed through the arrondissements of Paris. We distinguish arrondissements with a low level of social mix, both with a high average price and a low average price and less segregated arrondissements.

1 Introduction

The place where people live and the way they are distributed across cities matter, from both a social and an economic point of view. This article deals with the dynamics of price formation in a urban housing market and seeks to explain how individuals with different willingness to pay (WTP) are distributed over a city. Housing price formation can then entail segregation when people are heterogeneous.

The segregation we consider here is specifically related to the repartition of income over space. We deal with this issue via the modeling of transactions on the real estate market and by looking at the resulting spatial distribution of prices. From the modeling of the dynamics, we provide an analytical analysis of the stationary state, and the results are illustrated with numerical simulations. Following Von Thünen’s model and Alonso’s study (see [1]) of land use, we start from the assumption that, in the context of a monocentric city, the center is the most convenient place to live. We then show how competition between people affects the spatial distribution of agents who are heterogeneous in their WTP. We also give an outline of the results and theoretical analysis for more general (non monocentric) types of cities.

Seeking to describe how housing prices and agents of different incomes are distributed over space call for an assumption about people’s preferences. Do they prefer to live with people who are richer than they are, or poorer? In this article, we assume that individuals prefer to live with people who are richer than they are. This is in line with the literature showing that when people decide where to live, one of their main criteria is the quality of their environment, as
demonstrated in the literature on hedonic prices. We note however that the model allows for many generalizations, and in particular different hypothesis on social preferences could be studied within the same framework.

The model we propose here is inspired by [4] and [2], who model the evolution of the spatial distribution of crime in a city, attributing to each location an attractiveness for illegal activity (the more attractive a location the higher the probability of a burglary). In the present context of housing market, the originality of our model is precisely that each agent attributes to each location a specific level of attractiveness. This attractiveness results from a combination of an intrinsic or objective part, and of a subjective part. The endogenous (subjective) attractiveness results from the intrinsic individuals’ social preferences (the closer my fellows, the higher the level of subjective attractiveness). Then the main assumptions of the model are that, (i) people make decisions according to both their WTP and their individual evaluation of the level of attractiveness of the different locations; (ii) buyers, heterogeneous in their WTP, base their search for housing on the level of attractiveness of the location of the dwelling; (iii) agents are both buyers and sellers; (iv) the intrinsic attractiveness depends on the distance from the geographical center (maximum at the center and decreasing with the distance). This last assumption describes a monocentric city. Nevertheless, other cases, in particular polycentric cities, can be easily modeled by considering other forms of the intrinsic attractiveness as it will be shown.

This article gives an analytic demonstration of the main theoretical results, then extend the understanding of the dynamics through the simulation of an agent-based model. The pertinence of the results is then verified, through the analysis of a database of real-estate transactions in Paris.

2 Theoretical model: the assumptions

A model of residential location is proposed: people take their decision according both to their revenues and their individual evaluation of the level of attractiveness of the different locations. Buyers, heterogeneous by their WTP are looking for flats, according to the level of attractiveness of the place where the flat is located.

A0: Space

A finite number of goods (apartments for sale) are located on a discrete set $\Omega$ of locations $X$ uniformly distributed in a bounded open set $\tilde{\Omega}$ in $\mathbb{R}^2$. The total number of locations is $\text{Card}(\Omega) = L^2$, and the space is of linear size (diameter) $aL$, where $a$ gives the typical distance between two neighboring locations. There is a total number $N$ of apartments with identical intrinsic characteristics at each location $X$ in $\Omega$.

We consider Cartesian coordinates on the space $\tilde{\Omega}$, the origin being considered at the geographical center of this set. The distance $D(X)$ to the center of a location $X$ of coordinates $(x, y)$ is thus $D(X) = \sqrt{x^2 + y^2}$ with $D(X) \leq aL/2$. 
A1: Agents

Time is discrete and indexed by $t$. The time increment is $\delta t$. At each time step, there is a finite number of agents in the economy, who can be in one of the three following states: (1) buyer, (2) seller, (3) housed. We assume an infinite “reservoir” of agents outside the city. From this reservoir, at each time $t$ a constant number $\gamma L^2$ of agents arrive on the market. They are new buyers who add to the buyers who did not succeed in the previous period. Housed agents become sellers at a homogeneous rate $\alpha$. When they succeed in selling their good, they leave the market. Note that the total number of agents of each type, buyers, sellers and housed, are dynamical variables since they depend on the success rate of the previous period, and on the inflow and outflow rates.

A2: Demand prices

Agents are characterized by their willingness to pay (or sell) (hereafter WTP or WTS), which determines the maximum price the agent is ready to pay for an asset. For simplicity, we consider a finite number $K$ of WTP. Agents with the same WTP are designed by $k$-agents, $k \in \{0, ..., K-1\}$, and have WTP $P_k$, ordered by increasing values, $P_0 < P_1 < ... < P_{K-1}$. When the agent is acting as a buyer, his demand price is $P^d_k = P_k$. We assume that these $K$ values are uniformly distributed among the agents in the external reservoir.

A3: Attractiveness

The attractiveness of each location depends both on intrinsic objective characteristics and on subjective characteristics which depend on the social neighborhood. At each location $X$ corresponds an attractiveness which has two components:

1. an *intrinsic attractiveness*, $A^0(X)$, idiosyncratic to the position considered, that may be, for example, linked to the presence of amenities. It is assumed to be independent of time in a first approach. The space can be decomposed as the union of $n$ sub-spaces $\Omega^a, a = 1, ..., n$, such that, on each one of these sub-spaces, the attractiveness $A^0(X)$ is monotonously decreasing with the distance to the center of the city.

2. a *subjective attractiveness*, whose value depends on the WTP of the agent that looks at the location $X$. It is all the higher from the point of a view of a $k$-agent as agents of same and higher WTP are buying a good on $X$.

The total attractiveness at time $t$ on a location $X$ seen by a $k$-agent is updated at each step of the dynamics according to:

\[ A_k(X, t + \delta t) = A_k(X, t) + \omega \delta t \left( A^0(X) - A_k(X, t) \right) + \epsilon \delta t v_{k>} (X, t) \]  

(1)

with

\[ v_{k>} (X, t) = \sum_{k' \geq k} v_{k'} (X, t) \]  

(2)
where $v_k(X,t)$ is the density of $k$-buyers on location $X$ which realize a transaction at time $t$. In other terms, the attractiveness increases with the demand. Henceforth, what we mean by density is a number over the number of space locations. (i) When there is no transaction at a given location for a certain amount of time, the attractiveness relaxes towards its intrinsic value $A^0(X)$; and, (ii) agents prefer to have neighbors with at least equivalent WTP. Note that other hypotheses on agents’ preferences can be dealt with in this framework.

A4: Offer prices

Each seller is characterized by an offer price which depends on his WTS and on the intensity of the demand, through the level of the mean attractiveness of the location. A $k$-agent, when acting as a seller, has his WTS determined by his WTP $P_k$. The resulting offer price is given by

$$P^o_k(X) = P^0 + (1 - \exp(-\xi \bar{A}(X,t))) P_k, \quad \bar{A}(X,t) = \sum_{k=0}^{K-1} \frac{A_k(X,t)}{K}$$

where $P^0$ is the minimum price of an offer, $\xi$ is a parameter.

A5: The matching

At each time step (between times $t$ and $t + \delta t$):

1. The density $\rho_k$ of $k$-buyers is the sum of a constant proportion $\frac{\gamma}{K}$ of new buyers, and of the density of buyers who have not yet succeeded. Each one of these $\rho_k L^2$ agents has a probability $\pi_k(X,t)$ to visit a given location $X$, that depends on the attractiveness of the location:

$$\pi_k(X,t) = \frac{1 - \exp(-\lambda A_k(X,t))}{\sum_{X' \in \Omega} 1 - \exp(-\lambda A_k(X',t))}.$$ (4)

2. At location $X$, a transaction between a $k$-buyer and a $k'$-seller with offer price $P^o_{k'}(X) = P^0 + (1 - \exp(-\xi \bar{A}(X,t))) P_{k'}$, can be realized if $P_k > P^o_{k'}(X)$.

3 Theoretical model: The equilibrium

3.1 Continuous time dynamics

The evolution of the system is further formalized through partial differential equations, taking the continuous time limit (that is $\delta t \to 0$).

The total density of settles agents with the WTP $P_k$, $u_k(X,t)$, satisfies:

$$(1 - \alpha) \partial_t u_k(X,t) = v_k(X,t) - \alpha u_k(X,t)$$

(5)
At each period, a proportion $\alpha$ of people leaves the city.

The updating rule of the attractiveness of a location $X$ seen by a $k$-agent gives in the continuous limit:

$$\partial_t A_k(X,t) = \omega (A^0(X) - A_k(X,t)) + \epsilon v_{k_+}(X,t)$$  \hspace{1cm} (6)

The density of candidates $\rho_k(X,t)$ is written as

$$\rho_k(X,t) = v_k(X,t) + \bar{v}_k(X,t),$$ \hspace{1cm} (7)

with $\bar{v}_k(X,t)$ pointing out the density of agents who do not succeed in buying a $X$-good at time $t$. Given the rules of the dynamics, the evolution of the candidates density $\rho_k(X,t)$ can be written as

$$\partial_t \rho_k(X,t) = -\rho_k(X,t) + \frac{\gamma}{K} \pi_k(X,t) + \pi_k(X,t) \sum_{X' \in \Omega} \bar{v}_k(X',t)$$ \hspace{1cm} (8)

### 3.2 Equilibrium

#### Stationary state: the general case

All the variables in the stationary state will be written with a star $^*$. From equations (5), (6) and (8), one gets for the stationary state:

$$v_k^*(X) = \alpha u_k^*(X),$$ \hspace{1cm} (9)

$$A_k^*(X) = A^0(X) + \frac{\epsilon}{\omega} v_{k_+}^*(X)$$ \hspace{1cm} (10)

$$\rho_k^*(X) = \pi_k^*(X) \sum_{X' \in \Omega} \rho_k^*(X').$$ \hspace{1cm} (11)

In the stationary state, the total density of $k$-housed agents on the whole space is equal to $K \frac{\alpha}{\gamma}$. The total density of $k$-transactions during a step is $\frac{K}{\gamma}$. Thus, the total number of housed agents is the same for all the $k$ values.

In the following, we restrict the analysis (existence and characterization) of equilibria in a non-saturated regime, as defined below.

**Definition: non-saturated equilibrium.** A non-saturated equilibrium is defined as an equilibrium where, for any given $k \in \{0, \ldots, K-1\}$, at any location $X \in \Omega$, either the WTP of the $k$-agents are too low, so that none of them can afford to buy a good at this location, hence $v_k^*(X) = 0$, or the $k$-agents can afford to buy a good at this location, and in such case any $k$-demand is satisfied, that is

$$\bar{v}_k^*(X) = 0.$$ \hspace{1cm} (12)

Agents with the highest WTP succeed to live wherever they want, while it is not systematically the case for the agents with the lowest WTP. We will show below that, in the stationary state, there exists a WTP threshold $P_c^*$ such that the excess demand is null for agents with a WTP larger than $P_c^*$, whereas
agents with a lower WTP cannot settle at a distance lower than the threshold: we show that this threshold depends on the value of the WTP.

We will denote by $\bar{k}$ the marginal $k$ for which the WTP is equal to $P^*_c$ (hence $P^*_k = P^*_c$). This WTP threshold results from the dynamics, depending on the magnitude of the subjective contribution of the attractiveness, depending itself on the inhabitants on a location.

**Above the threshold**

Let us first assume $P^*_c \leq P_{K-1}$, hence $\bar{k} \leq K - 1$. We consider the $k$-agents with $k \geq \bar{k}$: they have high enough revenues to buy a flat wherever they want. We consider here the small $\lambda$ limit: we take thus a probability to choose a location

$$
\pi_k^*(X) = \frac{A^*_k(X)}{\sum_{X' \in \Omega} A^*_k(X')} = \frac{A^*_k(X)}{Z_k}.
$$

**Proposition 1:** In a non-saturated equilibrium, the density of housed $k$-agents with $k \geq \bar{k}$ does not depend on the level of individual WTP but only on the intrinsic attractiveness of the location, according to:

$$
\forall k \geq \bar{k} \quad \forall X \in \Omega, \quad u^*_k(X) = \frac{\gamma}{K^\alpha} \frac{A^0(X)}{Z^0}, \quad Z^0 = \sum_{X \in \Omega} A_0(X).
$$

**Below the threshold**

On the real estate market, agents with high WTP and agents with low WTP are in competition. The presence of agents with high WTP creates an area of high attractiveness and high prices. A minimum condition for transactions is that demand prices of the $k$-agents are higher than the offer prices of the $k$-agents.

From equation 3 one gets the condition:

$$
P_k \geq P^0 + (1 - \exp(-\xi \tilde{A}^*(X))) P_k
$$

This condition enables us to look for the set $\Omega_k$ where the $k$-demand is non null.

**Proposition 2:** In a non-saturated equilibrium, for each level $k$ of individual WTP such that $k < \bar{k}$, there is a restricted space $\Omega_k$ in which the $k$-agents can afford to buy the goods present. This space, $\Omega_k$, is the union of sub-spaces $\Omega^a_k \subset \Omega_k$, where the agents have a sufficiently high WTP to exchange. For $k < \bar{k}$, the density of housed $k$-agents depends on the level of individual revenues and on the intrinsic attractiveness of the location according to:

$$
u^*_k(X) = \frac{\gamma}{K^\alpha} \frac{A^0(X)}{\sum_{X' \in \Omega_k} A^0(X')} \text{ if } X \in \Omega_k \equiv \bigcup_{a=1}^n \Omega_k^a
$$

where

$$
\Omega_k^a \equiv \{ X \in \Omega^a | D(X) \geq d_k^a(k) \}
$$

where $d_k^a(k)$ is a critical distance (to the center) that determines the space $\Omega_k^a$, where a $k$-agents can theoretically buy a good. For each space $\Omega^a$, the $k$-agents can afford the goods in the locations $X \in \Omega^a$ for which $D(X) \geq d_k^a(k)$.
WTP Threshold

We assume that the WTP are uniformly distributed between two extreme values:

\[ P_k = P_0 + \Delta \frac{k}{K-1}, \quad k = 0, ..., K-1. \]  (18)

Let us first focus on the case of a monocentric city (hence \( n = 1 \)), where the intrinsic attractiveness decreases from the center on the whole space \( \Omega \), modeling here the preferences of the agents for the center:

\[ A^0(X) = A_{max}^0 \exp \left( -\frac{D(X)^2}{R^2} \right) \]  (19)

Consequently, for each \( k \) the restricted space \( \Omega_k \) is characterized by a single critical distance \( d_c^*(k) \) to the center, above which the \( k \)-agents’ demand is positive.

One finds that this critical distance \( d_c^*(k) \) satisfies:

\[ d_c^*(k) \geq R \sqrt{-\ln \left( \frac{\frac{3}{4}A_{max}^0}{1 + \frac{c^0_\gamma}{2\omega^0} \left( \frac{K+1}{K} - \frac{(k+1)\gamma}{K^2} \right)} \right)} \]  (20)

If the argument of the logarithm is smaller or equal to 1, \( d_c^*(k) = 0 \).

The WTP threshold, \( P_c^* \), is given by the smallest \( k \) value (which we have denoted \( \bar{k} \)) for which \( d_c^*(k) = 0 \). By taking the infinite limit for \( K \):

\[ P_c^* = \Lambda \exp \eta \left( 1 - \left( \frac{P_c^* - P_0}{\Delta} \right)^2 \right) \]  (21)

with \( \eta \equiv \frac{c^0_\gamma}{2\omega^0} \xi A_{max}^0 \) and \( \Lambda \equiv P_0 e^{\xi A_{max}^0} \).

If \( P_0 < \Lambda \exp \eta \), and \( \Lambda < P_0 + \Delta \), there exists a WTP threshold \( P_c^* \) in \([P_0, P_0 + \Delta]\). If \( P_0 > \Lambda \exp \eta \), no agent can be housed a good in the center.

As one can check on Eq. (21), the threshold \( P_c^* \) is an increasing function of \( \eta \): this is reasonable, since it means that \( P_c^* \) increases if the incoming flux increases, or if the social factor \( \epsilon \) increases.

Generalization: Consider the more general case \( n > 1 \) (more general than the purely monocentric city case), where the space \( \Omega \) is the union of disjoint sub-sets \( \Omega^a \) \((a = 1, ..., n)\) on which the attractiveness is monotonous. Then, there exists a critical distance \( d_c^a(k) \) for each sub-space \( \Omega^a \).

4 Numerical simulations

Numerical simulations have been performed on a square lattice of linear size \( L = 100 \), hence with \( L \times L = 10000 \) sites. The origin is the center of the lattice. The intrinsic attractiveness \( A^0(X) \) is chosen as in (19). The initial attractiveness of a location \( X \) is taken to be equal to the intrinsic attractiveness.

The dynamics used for the numerical simulations is the following:
• Agents arrive in the city,
• they candidate on a location according to its attractivity,
• at the site level
  – as long as there is any left, an offer and a candidate that can afford it are randomly selected and matched,
  – attractivity is updated,
• a fraction $\alpha$ of the housed agents leave the city.

The numerical simulations are performed with the following values of the parameters: the total number of offers $N$ is chosen identical on each of one of the 10000 sites, $N = 200$; $\lambda = 10^{-2}$, $\xi = 0.1$, $A^0_{\text{max}} = 1$, $\epsilon = 0.022L^2$, $R = 10$, $\Delta = 225000$, $\omega = \frac{1}{10}$, $P^1 = 200000$, $\frac{\gamma}{\kappa} = \frac{1000}{L^2}$, $\beta = 0.1$. The initial number of offers is voluntarily high enough to prevent the system from saturating.

We mainly focus on the spatial distribution of housed people in the stationary state. In particular we compare the simulated and theoretical spatial distributions of the housed agents per WTP. The theoretical results have been obtained by considering a non-saturated regime in which there only exist two states of occupation on a site: either all the potential buyers from a given WTP manage to be housed, or no agent from a given WTP can be housed.

4.1 Socio-spatial segregation and income mix

The dynamics leads to a non-uniform distribution of the $k$-agents on each location. This suggests that the level of social diversity is weakened by the dynamics of social preferences and the unequal income distribution. Fig. 1 shows the occupancy ratio for the different WTP in the stationary state with respect to the distance to the center.

**Figure 1:** Occupancy ratio per WTP versus the distance to the center in the stationary state ($K = 10$).

**Figure 2:** Occupancy number with respect to the distance to the center for the WTP $k = 9$ ($K = 10$).
The simulated city displays three distinguishable zones (Fig. 1):

1. The central zone where the main inhabitants are the agents from superior revenue, i.e. with high reservation prices.
2. The peripheral zone where the main inhabitants are the agents from inferior WTP, i.e. with low reservation prices.
3. The intermediary zone (distance to the center close to 20) where the agents from all WTP are present in comparable proportions, what one calls social mix in what follows.

The presence of this zone of social mix was not conspicuous, since it could have been expected that agents could be segregated by WTP on the whole lattice.

Above the threshold Fig. 2 represents the occupancy number obtained with $K = 10$ WTP for $k = 9$ (the higher WTP) with the simulations and with the analytical formula (Eq.(15)). The density obtained analytically turns out to be a good approximation for the occupation ratio of the agents that can settle at the center (i.e. $d^*_c(k) = 0$).

![Figure 3: Evolution of the critical distance with respect to $k$ ($K = 50$).](image1)

![Figure 4: Occupancy number versus the distance to the center for $k = 2$ ($K = 10$).](image2)

Critical distance The critical distances (Fig. 3) shows a good agreement between the simulations and the analytical formula for the low values of $k$.

Below the threshold Fig.4 depicts the occupancy number with respect to the distance to the center for $k$-agents non housed in the center ($d^*_c(k) > 0$). A good concordance between simulation and analysis is observed.

4.2 Social mix index

To evaluate the level of social diversity, we introduce a measure of segregation derived from the dissimilarity index proposed by Duncan and Duncan (1955):

\[
ID(X) = \sum_{k=0}^{K-1} \left| \nu_k(X) - \frac{1}{K} \right|, \quad \nu_k(X) = \frac{u_k(X)}{\sum_{k=0}^{K-1} u_k(X)}.
\] (22)
This is the difference between the uniform and effective distributions of agents. The greater this index is, the greater the segregation between agents.

An alternative choice is the mixing entropy

$$H(X) = -\sum_{k=0}^{K-1} \nu_k(X) \log \nu_k(X)$$  \hspace{1cm} (23)

One computes these quantities for the stationary state in the case (a) (ordered offers) for two different rates of newcomers $\gamma_1 = \gamma$ and $\gamma_2 = 3\gamma$ (Fig. 5).

The relatively low entropy in the intermediate zone confirms the presence of an area of social mix. When the rate of newcomers increases, the index (resp. entropy) is on the whole higher (lower) meaning a general decline of social mix.

5 Empirical evidence

This section proposes an empirical validation of the results analytically demonstrated and already validated by simulation. The question here is to understand how much the prices distribution in Paris can be explained by the phenomena described above. The geometrical form of Paris gives a special importance to the geographical center. In this configuration, it can be interesting to verify the importance of the position of a dwelling in the formation of its prices.

The main information source on the real estate prices in Paris is the B.I.E.N. database, organized by the “Chambre des Notaires de Paris” which registers real estate transactions for Paris and Ile-de-France. We restrict to the year 1994 to compare simulated and empirical static trends. During this period, about 13000 Parisian transactions were registered. The average price of a flat is (in euros) 143300, the standard deviation is around 90000 and the distribution is clearly not normal (the normalized kurtosis coefficient is strictly positive).
5.1 A particular Parisian dynamics

The empirical analysis of the dynamics of price formation in Paris suggests that prices are moving according to a double dynamics which could be illustrated through our modeling of attractiveness (extrinsic and intrinsic). The evolution of average prices per arrondissement exhibits a particular spatiotemporal dynamics (Fig. 6). In all the arrondissements, prices are going down from 1990 until 1995 then up. But on the increasing part of the dynamics, the ranking of the arrondissement (from the highest price to the lowest one) has changed.

Looking at the map of the prices for the year 1994 on Fig. 7, a first evidence is that prices are high in the center. As a first step, one may look at the average price as function of the distance to the center. Quite remarkably, there is a marked trend for a decrease of prices from the center, so that the monocentric model can be seen as a first order approximation of the Parisian market. However, there are obvious deviations from this global trend. In particular, the “Rive Gauche” (left bank) effect is important: at a similar distance, prices are higher on the left bank than on the right one. In addition, prices are high in the 16th arrondissement at the very periphery of Paris, giving a “hot-spot” of high prices far from the center.

**Data inspired modeling:** As we have seen, the pure monocentric model is not sufficient to account for the Parisian market. Hence, we consider a more specific model of the city, which combines a general preference for the center together with preferences for some local particularities.

First, instead of using a square lattice in the simulations, we use a stylized map of Paris. It consists of three concentric zones of radius $R_1$, $R_2$, $R_3$ each zone with respectively 4, 7 and 9 areas representing the arrondissements (Fig. 8). Second, we assign to each one of these “arrondissements” a specific intrinsic attractiveness, decreasing with respect to the city center. Consequently, the modeled city is a particular case of the model analytically studied in the previous
sections, with \( n = 20 \) sub-spaces \( \Omega^a, a = 1, \ldots, 20 \). The intrinsic attractiveness parameters are determined for each modeled arrondissement according to the averages of transaction prices of the corresponding (real) arrondissement in the year 1994. The intrinsic attractiveness of each arrondissement \( a \) is thus given by:

\[
A^{0,a}(X) = \left( \frac{P_{a ref}}{P_{a max ref}} \right)^2 \exp \left( -\frac{(R_a - D(X))^2}{R} \right)
\]  

where \( D(X) \) is the distance to the center, \( R_a \) is the shortest distance of the arrondissement to the center and \( P_{a ref} \) is the mean transaction price of the arrondissement \( a \). This form of the attractiveness takes into account the fact that some arrondissements are expensive even if they are far from the center of the city. The value of \( R \) has been chosen to be in the same order of magnitude as the distances to the center in the lattice used. The map of the attractiveness is represented on the right of the Fig. 8.

The order of magnitude of the other model parameters are chosen making use of data published by the INSEE (France’s National Institute of Statistics and Economic Studies). According to these data, there are about 375000 dwelling owners in Paris (we only consider the main homes), and of order 13000 transactions in one year. We consequently deduce a rate of moves of about 3.5%. In order to have transaction prices in the model comparable with the empirical ones, we take \( K = 13 \) different WTP with \( \gamma = \frac{1000}{L^2}, \alpha = 0.035 \) and \( \epsilon = 0.18L^2 \).

**Dissimilarity indices** We seek here to evaluate the level of segregation by the distribution of incomes over space. We thus compute the dissimilarity index as in section §4.2 and we consider the averages of the indices on each modeled arrondissement. Since the real estate base does not contain the incomes of the buyers, we use the transaction prices as a proxy for income distributions. Considering the year 1994, we classify the transaction prices and divide the prices intervals in sets of about 1000 transactions. We postulate that transactions
achieved in a same price interval were made by individuals belonging to a same income class. In other terms, each set of the transaction prices in the data correspond to transactions made by a given WTP in the model.

According to Fig. 9, the dissimilarity index of the modeled city shows variations through space comparable to those observed through the arrondissements of Paris. In particular, the weakest social mix (that is the highest values of the dissimilarity index) are observed in the 8, 16, 18, 19, 20-th arrondissements, both in the simulations and in the data. In reality, the arrondissements 8 and 16 corresponds to the rich arrondissements in Paris (the places where people’s WTP are above the threshold) while the arrondissements 18, 19, 20 correspond to the poorer places. Therefore the largest social mix appears in the mid-range arrondissements.

6 Conclusion

This paper studies a particular housing market model of dynamic prices formation and agents localization, through the distribution of income over the space. This actually introduces a new framework for studying such markets and the resulting socio-spatial segregation. The model allows to specify both the intrinsic attractiveness of a location, resulting for example from the amenities, and a subjective part, depending on the social preferences of the agents - and thus evolving in time together with the social characteristics of the neighborhoods.

For the particular choices made in the present study - a monocentric city and a preference for living with richer people - the analysis yields two results. First, as expected, a socio-spatial segregation appears, with richer people living near the center and poorer people at the periphery. Second, whatever the parameters of the simulation, it exists an area of social mix, between the two segregated zones. The fact that we consider agents heterogeneous in their WTP allow us to go beyond the simple Schelling case with binary status. Looking at the empirical distribution of prices in Paris for the year 1994, we observe that the simple Alonso model (prices higher at the center) is, as a first order approximation, a good description of Paris structure: higher prices are indeed concentrated around Notre Dame cathedral. The monocentric characterization of the housing market in Paris is correct in average but miss some specificities as the difference between prices on the left bank side (higher) and on the right bank side (lower). A better representation of the Paris housing market is then proposed by considering an intrinsic attractiveness specific to each one of the 20 arrondissements. One of the results is that we distinguish arrondissements with a low level of social mix, for the arrondissements with the highest average prices and for those with the lowest average prices, and less segregated arrondissements for the arrondissements with intermediate average prices. Taking the empirical price distribution as proxy for the income distribution, the simulated results compare well with the real estate data.

Future works will explore alternative hypothesis concerning the social preferences, such as agents preferring to live in more popular area, as well as concern-
ing the structure of the intrinsic attractiveness, such as shapes corresponding to polycentric cities. A further step will consist in analyzing the conditions on the dynamics of the subjective attractiveness for the emergence of “hot spots” - local domains of high prices unrelated with the level of the intrinsic attractiveness. This will go with the study of extensions of the model, notably, in the line of [4] and [2], adding a diffusion term in the attractiveness dynamics: a high attractiveness of a location is expected to have some positive influence on the attractiveness of nearby locations. It is such extension which is likely to lead to dynamical instabilities, with the emergence of hot spots of high prices, that could, for example, explain the changes in the ranking of the arrondissement by prices along the years.

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