The Power-Law Hypothesis for Limit Order Placements in the KOSPI 200 Futures Market

Seung Oh HAN, ChongSeok HYUN and Hyeng Keun KOO

Graduate Department of Financial Engineering,
Ajou University, Suwon, 443-749, Korea
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Abstract

We test the power-law hypothesis in the KOSPI 200 futures market for limit order placements using the method proposed by Clauset, Shalizi and Newman (2009[3]). With the order data submitted in the market between March 17, 2003 and March 11, 2004, we find that limit order placements rarely pass the power-law hypothesis. This contrasts to the most of empirical results reported in other markets until 2011.

We also find a strong evidence that supports the asymmetric behavior between ask orders and bid orders. The test results for subsamples divided by trading times show smaller rejection ratios than for the whole trading day sample. This suggestss that order placement behavior is time-varying during a trading day.

Keywords: fat-tail event, high-frequency data, KOSPI 200 futures, limit order, market microstructure, maximum-likelihood estimation, order book, power law

*Electronic address: seungohan@gmail.com
†Electronic address: ordeeq@ajou.ac.kr; Corresponding author; Fax: +82-31-219-3664
‡Electronic address: koo_h@hanmail.net
I. INTRODUCTION

Power-law distributions arise naturally from economics and finance as well as from biology, computer science, and sociology (Gabaix (2009[6]), Mitzenmacher (2004[16], 2006[17])). Such phenomena have, in particular, been reported in financial markets: asset prices, trading volumes, numbers of transactions observe power law distributions (see, e.g., Plerou et al. (1999[19], Gabaix et al. (2003[7]), Zovko and Farmer (2002[24]), Bouchaud et al. (2002[2]), Potter and Bouchaud (2003[21]), Farmer and Lillo (2004[4]), Plerou et al. (2004[20]), Farmer et al. (2004[5])). Inspired by this observation the power-law hypothesis proposes that a random variable is distributed by a power-law distribution. The hypothesis provides a useful benchmark when a researcher studies a data set with a large number of extreme observations.

In this paper we test the power-law hypothesis for limit order placements in the KOSPI 200 futures market in Korea. The KOSPI 200 futures market has been the most liquid futures market since 2002 and has an open limit order book. In contrast to the previous research on stock exchanges (London and Paris) we reject the power-law hypothesis for the limit order placements in the KOSPI 200 futures market.

Zovko and Farmer (2002[24]) found that the traders in the London Stock Exchange place limit orders in the way the unconditional cumulative distribution of relative limit prices with respect to the best available price decays roughly as a power law. Bouchaud et al. (2002[2]) made a similar observation for the Paris Bourse. Subsequent works have made attempts to give theoretical justifications to the empirical findings and others have tried to verify the power law for other markets (Potters and Bouchaud (2003[21]), Lillo (2007[14]), Zovko and Farmer (2007[25]), Mike and Farmer (2008[18])). Clauset et al. (2009[3]), however, have criticized the empirical results on the ground that most of the results are based on regression models and are subject to significant systematic errors. Furthermore, they have proposed a statistically objective and consistent approach that combines maximum-likelihood fitting methods with goodness-of-fit tests based on the Kolmogorov-Smirnov statistic and likelihood

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1 Up to our knowledge this is the first paper which tests the power-law hypothesis for the limit order placements in the KOSPI 200 futures market. Closest to ours is the work by Lee et al. (2011[13]) who have examined the volume of orders and its impact on the KOSPI 200 futures market.

2 World Federation of Exchanges (http://www.world-exchanges.org/).
ratios. We adopt their approach and test the power-law hypothesis for order placements in the KOSPI 200 futures market and reject the hypothesis.\footnote{We have used the code set provided by Clauset et al. (2009\cite{3}) which can be freely downloaded from http://tuvalu.santafe.edu/~aaronc/powerlaws.}

There has been an active debate whether the power-law hypothesis is valid for returns on stocks (Gabaix et al. (2003\cite{4}), Farmer and Lillo (2004\cite{4}), and Plerou et al. (2004\cite{20})). In this paper we contribute to the literature by rejecting the hypothesis for limit order placements in the KOSPI 200 futures market.

Another contribution of ours is that we use a large data set consisting of limit order placements from March 17, 2003 to March 11, 2004.\footnote{The number of order placements in a day is on average approximately equal to 100,000 in the KOSPI 200 futures market. See TABLE \ref{}.} Previous researchers have mostly used data sets consisting of order placements over several days, not exceeding one month. Our large data set allows us to conduct more robust hypothesis tests.

The remainder of the paper is organized as follows. We state the power-law hypothesis for order placements in the KOSPI 200 futures market in Section 2. Section 3 describes the data and section 4 reports our empirical findings. We conclude and discuss a potential direction for future research in the last section.

II. THE POWER-LAW HYPOTHESIS FOR ORDER PLACEMENTS IN THE KOSPI 200 FUTURES MARKET

Let us denote the best ask price and the best bid price at time $t$ by $a(t)$ and $b(t)$, respectively. We consider the probabilities of order placements at the limit ask price $a(t) + \Delta$ and at the limit bid price $b(t) - \Delta$, where $\Delta$ is measured in tick sizes. Let the probability density of the former by $p_a(\Delta)$ and the probability density of the latter by $p_b(\Delta)$, respectively. The power-law hypothesis for order placements proposes that $p_a(\Delta)$ and $p_b(\Delta)$ are drawn from the following probability distributions:

$$ p_i(\Delta) \propto \Delta^{\mu_i} \quad \text{for} \quad i = a, b, \tag{1} $$

where $\mu_a/\mu_b$ is the exponent of power law for incoming limit ask/bid order placements. Further, we assume that or $i = a, b$, there exists some $\Delta_i$ and the power-law is applied
only to limit orders satisfying $\Delta \geq \Delta_i$. With this assumption, the power-law hypothesis (1) means that, for each $i = a, b$, limit order placements follow the following:

$$p_i(\Delta) = \frac{\mu_i - 1}{\Delta_i} \left( \frac{\Delta}{\Delta_i} \right)^{-\mu_i} \text{ for } \Delta \geq \Delta_i. \quad (2)$$

Let us consider also the complementary cumulative distribution function of a power-law distributed variable. We define $P_a(\Delta) = Pr(x \geq a(t) + \Delta)$ for ask orders and define $P_b(\Delta) = Pr(x \leq b(t) - \Delta)$ for bid orders, where $x$ denote the limit price of a limit order. Then the power-law hypothesis in Equation 2 is equivalent to the linear relationship between $\log P_i(\Delta)$ and $\log \Delta$ for each $i = a, b$.

Clauset et al. (2009[3]) have proposed a testing method for power-law hypothesis that combines maximum-likelihood fitting methods with goodness-of-fit tests based on the Kolmogorov-Smirnov statistic and likelihood ratios. The method also enables us to estimate $\Delta_a$ and $\Delta_b$.

III. DATASET

The KOSPI 200 futures is a derivative product based on the KOSPI 200 index which is the weighted average of the largest 200 companies in market capitalization listed in the Korea Exchange (KRX). The unit size of the futures contract is KRW 500,000. The KOSPI 200 futures expire in March, June, September, and December, and the expiration day is the second Thursday of the expiration month. The market opens at 8:00 AM and closes at 15:15 PM, except that it opens at 8:00 AM and closes at 14:50 PM on the last trading day. It has two types of auctions – the single price auction and the continuous auction. The first daily single price auction takes place at 9:00 AM with orders accumulated from 8:00 AM to 9:00 AM. The second daily single price auction occurs at 15:15 PM with orders accumulated between 15:05 PM and 15:15 PM, but there is no second single price auction on the last trading day. The continuous auction starts at 9:00 AM and ends at 15:05 PM, except on the last trading day when it starts at 9:00 AM and ends at 14:50 PM. It operates fully electronically with the tick size of 0.05 point, which amounts to KRW 25,000 per unit.

As Stoev et al. (2011[23]) argue the common methods of choosing the lower bounds visually on a log-log plot of the probability distribution function or the complementary cumulative distribution function are too subjective and can be sensitive to noise or fluctuations in the tail of the distribution.
contract. The daily price limit is $\pm 10\%$ of the base price, usually the closing price of the previous trading day.

We have selected the orders for the KOSPI 200 futures contracts expiring in the closest months between March 17, 2003 and March 11, 2004 (including 240 business days) from the IFB Futures Transaction Database provided by the Institute of Finance and Banking of Seoul National University.$^6$ TABLE I summarizes the data set we have used in this paper.

The IFB Database contains information for a transaction including its submission time, price, volume, the revision time if it was revised, and the cancellation time if it was cancelled. We have reconstructed the order book for continuous auctions from the database based on the transaction matching rule adopted and publicized by the Korean Exchange. We exclude marketable limit orders from the limit order book, since we would like to study only orders which provide liquidity, not those which consume liquidity.$^7$

**IV. EMPIRICAL RESULTS**

**A. The Overall Test Results**

TABLE II shows the test results for the power-law hypothesis. Surprisingly, we reject the power-law hypothesis for a very large proportion of sample days. With ask orders, we reject it for about 85.42\% of the total trading days, while we reject it for 96.25\% with bid orders.

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6 The sample period is chosen to span the most recent 1 year available in the database.

7 Marketable orders are buying orders with a price at or above the best ask price or selling orders with a price at or below the best bid price. Gu et al. (2008) have studied distributions of limit orders including marketable limit orders.
TABLE II: The number and ratio of rejection of the power-law hypothesis. The rejection ratio (figures in parentheses) is measured by the number of dates when the hypothesis is rejected divide by the total number of sampling dates.

In determining the plausibility of the power-law hypothesis, we use the goodness-of-fit test measure proposed by Clauset et al. (2009[3]). Following their recommendation, the criterion for rejection is the p-value measure being less than 10%.

FIG. 1 shows the log-log plot for the futures contract expiring in December 2003, and the sampling dates are between September 15 and December 11.\(^8\) In the figure the order placements appear to follow a power-law distribution, however, the humped shape at the far tail prevents them from passing the power-law hypothesis test.

The above observation is made clearer by looking at FIG. 2. It plots in the log-log scale the distribution for the whole orders for the futures contract expiring in December 2003. It also plots the distribution for the orders that are not rejected by the power-law test. Each blue solid line represents the distribution for a single day which is not rejected by the test. Note that the blue solid lines usually lie below the circled line (the plot of the distribution of the whole orders) and do not show the humped shape described above.

FIG. 3 shows estimated exponents in the power-law model for whole sample dates. The exponent usually ranges between 2.5 and 3.0, which is quite similar to the value reported by Lillo et al. (2005[15]). They show that the distribution of block trades is consistent with the power-law hypothesis with an exponent close to 2.5, whereas the distribution of order book trades is not consistent with the half-quantic law, and instead has a much thinner tail (see also Farmer and Lillo (2004[4]) and Plereu et al. (2004[20])). We get the average exponent for ask orders equal to 2.7085, and the average exponent for bid orders equal to 2.5189.

\(^8\) Even though we do not report here, we have obtained similar figures for other futures contracts.
FIG. 1: Log-log plot for the futures contract expiring in December 2003. The figure in the upper panel is for ask orders, and the figure in the lower panel is for bid orders. The sampling dates are listed in TABLE I.

FIG. 3 show that ask-side order placements and bid-side order placements show different behavior. Indeed the t-statistic for the test that $\mu_a > \mu_b$ is 9.7178, which strongly confirms that the exponents for ask orders are on average larger than those for bid orders. Even though the power-law is rejected in KOSPI 200 futures market, the statistic confirms the difference in behaviors of order placements exhibited by FIG. 3. TABLE III shows that the result is robust for all sub-sample periods. This can be interpreted that traders are more impatient when they want to clear the long position (or take the short position) than when they want to clear the short position (or take the long position) in the KOSPI 200
FIG. 2: Log-log plot for the whole orders of the futures contract expiring in December 2003 (circles), and for the orders that are not rejected by the power-law test (blue solid lines). Each blue solid line represents the distribution for a single day.

futures market. The result can be compared to Gu et al. (2008[8]), who have investigated the empirical regularities of order placements in the Shenzhen Stock Exchange and have reported asymmetric behavior between ask orders and bid orders. They have, however, obtained the asymmetric behavior not for the data of non-marketable limit orders but for the data consisting of both non-marketable limit orders and marketable limit orders. Here we obtain the result for the data consisting of only non-marketable limit orders.

We now report the lower bound, $\Delta$, that fits best for the power-law hypothesis.\footnote{According to Zovko and Farmer (2002[24]), $\mu$ represents patience of investors.}

\footnote{The lower bound was conventionally chosen arbitrarily in previous works (see e.g., Farmer et al. (2004[5])).} FIG. 4

\[ \Delta \]
FIG. 3: Estimated exponents in the power-law model.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0 : \mu_a \leq \mu_b$</td>
<td>3.575 (0.000)</td>
<td>5.779 (0.000)</td>
<td>4.498 (0.000)</td>
<td>6.120 (0.000)</td>
</tr>
<tr>
<td>$H_1 : \mu_a &gt; \mu_b$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE III: T-statistics for the test that $\mu_a > \mu_b$. P-values are in parentheses.

shows estimated values of the lower bound for the whole sample dates following the method based on the Kolmogorov-Smirnov statistic described in Clauset et al. (2009[3]). The value usually ranges between 4 and 10. This is intuitive in the sense that the KOSPI 200 futures market publicly shows only the best 5 quotations.

B. Tests for Different Trading Times

It is well documented that intraday trading commonly shows a U-shaped pattern in its volume (see e.g., Harris (1986[10])). Namely, heavy trading at the beginning and at the end of a trading day and relatively light trading in the middle of the day is very typical in financial markets.\(^{11}\) Khil and Chung (2005[12]) has reported that the U-shaped pattern is true also for the Korean stock market. Following such an observation, we divide the sample into three groups. The first group is the set of orders placed between 9:00 AM and 11:00

\(^{11}\) A theoretical justification for the U-shaped pattern has been provided by Admati and Pfleiderer (1988[1]).
FIG. 4: Estimated lower bound $\Delta$

AM; the second group for orders between 11:00 AM and 13:00 PM; and the third group for orders between 13:00 PM and 15:05 PM. We test the power-law hypothesis for these three groups.

TABLE IV shows the test results, where we get relatively smaller rejection ratios for each group than in the total sample. Note that the test results in TABLE II show the average rejection ratio of 85.42% for ask orders and that of 96.25% for bid orders. Remarkably, the rejection ratios for orders occurring between 11:00 AM and 13:00 PM are reduced to 70.42% for ask orders and to 73.75% for bid orders. This suggests that the order placement behavior is time-varying within a trading day.

TABLE V shows estimated exponents for the power-law distribution in different trading times. The exponents tend to increase with time in a trading day. This means that traders in the KOSPI 200 futures market tend to be less patient in the afternoon than in the morning. This seems intuitive in the sense that many traders who take positions in the morning would like to clear some or all of them in the afternoon, hence they behave more aggressively or less patiently.

The result in TABLE V also suggests that the time-varying nature of the power-law exponent may be a reason why we have relatively low rejection ratios in each trading time than in a whole trading day.
<table>
<thead>
<tr>
<th>Order Type</th>
<th>Trading Time</th>
<th>2003-JUN</th>
<th>2003-SEP</th>
<th>2003-DEC</th>
<th>2004-MAR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ask</td>
<td>9:00 AM – 11:00 AM</td>
<td>44</td>
<td>42</td>
<td>43</td>
<td>43</td>
<td>172</td>
</tr>
<tr>
<td></td>
<td>(73.33%)</td>
<td>(70.00%)</td>
<td>(68.25%)</td>
<td>(75.44%)</td>
<td>(71.67%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11:00 AM – 13:00 PM</td>
<td>46</td>
<td>37</td>
<td>41</td>
<td>45</td>
<td>169</td>
</tr>
<tr>
<td></td>
<td>(76.67%)</td>
<td>(61.67%)</td>
<td>(65.08%)</td>
<td>(78.95%)</td>
<td>(70.42%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13:00 PM – 15:05 PM</td>
<td>46</td>
<td>49</td>
<td>50</td>
<td>51</td>
<td>196</td>
</tr>
<tr>
<td></td>
<td>(76.67%)</td>
<td>(81.67%)</td>
<td>(79.37%)</td>
<td>(89.47%)</td>
<td>(81.67%)</td>
<td></td>
</tr>
</tbody>
</table>

TABLE IV: The number and ratio of rejection for the power-law hypothesis in different trading times. Rejection ratios, defined in the same way as in TABLE II, are in parentheses.

<table>
<thead>
<tr>
<th>Order Type</th>
<th>Trading Time</th>
<th>2003-JUN</th>
<th>2003-SEP</th>
<th>2003-DEC</th>
<th>2004-MAR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
<td>9:00 AM – 11:00 AM</td>
<td>46</td>
<td>45</td>
<td>57</td>
<td>52</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>(76.67%)</td>
<td>(75.00%)</td>
<td>(90.48%)</td>
<td>(91.23%)</td>
<td>(83.33%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11:00 AM – 13:00 PM</td>
<td>43</td>
<td>45</td>
<td>43</td>
<td>46</td>
<td>177</td>
</tr>
<tr>
<td></td>
<td>(71.67%)</td>
<td>(75.00%)</td>
<td>(68.25%)</td>
<td>(80.70%)</td>
<td>(73.75%)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13:00 PM – 15:05 PM</td>
<td>47</td>
<td>43</td>
<td>59</td>
<td>54</td>
<td>203</td>
</tr>
<tr>
<td></td>
<td>(78.33%)</td>
<td>(71.67%)</td>
<td>(93.65%)</td>
<td>(94.74%)</td>
<td>(84.58%)</td>
<td></td>
</tr>
</tbody>
</table>

TABLE V: Estimated exponents for the power-law hypothesis in different trading times.

V. CONCLUSION

We have tested the power-law hypothesis in the KOSPI 200 Futures market for limit order placements using the method proposed by Clauset et al. (2009[3]). With the order data
submitted in the market between March 17, 2003 and March 11, 2004, we have found that limit order placements rarely pass the power-law hypothesis. But estimation results for the exponent and the lower bound that fit the power-law distribution best are quite robust: the exponent is between 2.5 and 3.0 and the lower bound between 5 and 10.

We have also found a strong evidence that supports the asymmetric behavior between ask orders and bid orders. The test results for subsamples divided by trading times show smaller rejection ratios than for the whole trading day sample. This suggests that order placement behavior is time-varying during a trading day.

So far we have not taken into consideration the price limits in the KOSPI 200 futures market. We will need to analyze their effects on the power-law hypothesis by imposing appropriate sampling criteria.

Acknowledgments

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