Abstract

Previous analyses of macroeconomic imbalances have employed models that either focus exclusively on real-side effects or financial-side disturbances. Structuralist models make the highly unrealistic assumption that financial surplus firms effortlessly and costlessly transfer those surpluses to deficit firms, which require additional savings to sustain their plans for capital accumulation. On the other hand, there exists a well-developed, rigorous and elegant literature that uses the multi-agent systems (MAS) approach to analyze the recent financial crisis. This literature focuses exclusively on the financial sector to the neglect of the real economy. In this paper, we build on the MAS model of Setterfield and Budd (2008), relaxing some of the restrictive assumptions regarding the goods market in that model, and adding a financial sector. The latter is inspired by the financial models of Johansen et al. (2000), Voit (2005), Vandewalle et al. (1999) and others. The result is a robust model of the economy in which the real and financial sectors are integrated and interact with one another. The contribution is to show that once a grid architecture based on preferential attachment of financial agents is reinforced by real-side flows of savings and investment, the potential for financial instability increases.

1 Introduction

The motivation for this paper is the “great immoderation”, the rise in the number of significant stock-price drawdowns over the last three decades. Financial crises in various guises are usefully surveyed in Reinhart and Rogo (2009), who identify the build up of debt, either public or private, as the principal cause of financial collapse. The paper employs a multi-agent systems (MAS) model to study the effects of adding a real sector to a relatively well-developed and now standard analysis of the financial system that arises out of network analysis (Acemoglu et al., 2011). Our model builds on an existing real-side MAS model, due to Setterfield and Budd (2008), which has roots in the structuralist tradition (see Taylor (1983), Taylor (1991), and Setterfield (2010)). A financial sector, inspired by the MAS financial models of Johansen et al. (2000), Voit (2005), Vandewalle et al. (1999) and others, is then added. The result is a model of the economy in which the real and financial sectors are integrated and interact recursively. Specifically, financial intermediation can constrain investment spending by firms, and hence the pace of growth in the real sector. Meanwhile, the profits and savings generated in the real sector affect the ability and willingness of financial intermediaries to lend. The contribution is to show that once a grid architecture based on preferential attachment of financial agents is reinforced by real-side flows of savings and investment, the potential for financial instability increases.

The paper is organized as follows. The next section discusses a simplified two-sector real-side model that shows how surplus savings of some firms are transferred to other firms that wish to investment more than they have saved. The section also explains how the aggregate demand sharing mechanism is implemented in the more disaggregated MAS model. The third section provides the details of how the financial agents interact with the real side, with emphasis on the network structure of the financial system. This section discusses preferential attachment and shows how the special nature of the structuralist model, augmented by a financial system provides an appropriate testbed for the study of real-financial interactions that can...
lead to a crash. The fourth section presents the simulation design and results, with attention to how crashes are identified. A typology of crashes is offered that traces their roots to whether money is fully endogenous as in the simple structuralist model and then to the details of the network structure. Simulation results are then discussed and finally statistical significance is established. It is seen that preferential attachment, combined with a simple real side, can enhance the ability of the financial system to channel surplus savings to investment. It can also produce bouts of significant financial instability, however. An appendix contains the pseudo code of the model.

2 The real side

The set of agents is divided into two subsets, firms and traders. A firm is defined as a production process that generates output and employs labor. Each firm is assumed to operate only one process. The financial intermediaries of the model are the traders and serve to channel firm savings into firm investment for the system as a whole. Traders have firms as clients. Each firm must have at least one trader, but may have more than one. Each trader has one and only one client, the firm to which that trader is assigned in the setup of the model.

The real side of the model is derivative of Setterfield and Budd (2008), in which a standard structuralist model is recast as a multi-agent system. They treat each firm as a separate economy with its own growth dynamics, essentially as in a trade model. Here a second approach is adopted, in which each firm operates a productive process within a macroeconomy, thereby sharing in available aggregate demand at a given instant in time.

In most structuralist models the level of investment determines the amount of savings in the system through the multiplier process and money is fully endogenous. These two characteristics of the model are more closely related than is sometimes recognized. Assume for the moment that only firms save while households spend all their income. Unless each firm finances its investment out of its own profits, some borrowing and lending from other firms must take place. The standard structuralist model must, therefore, undergo substantial modification to enable an analysis of financial collapse.

Under the usual assumptions that labor income per unit of output, \( l_i \), is spent on consumption while a fraction, \( s_i \), of capitalists’ income, \( \pi_i = 1 - l_i \), is saved, aggregate demand in period \( t \), \( Y_t \), can be written as

\[
Y_t = \sum_j (1 - s_j \pi_j) x_{jt} + I_{jt-1}
\]

where \( I_j = g_j(u_j)K_j \) is investment by the \( j \)th firm. Here \( K \) denotes the capital stock and \( g(u) \) is an accumulation function that depends on capacity utilization and animal spirits as in the standard structuralist vernacular. Output, \( x_{it} \) for the \( i \)th firm can then be written

\[
x_{it} = \theta_i Y_i
\]

where the share, \( \theta_i \), of aggregate demand absorbed by the \( i \)th firm is proportional to the capacity, \( Q_i \), of each firm

\[
\theta_i = \frac{Q_i}{\sum_j Q_j}
\]

With capacity utilization, \( u_{it} = x_{it}/Q_{it-1} \), and assuming a constant capital-output ratio, \( v_i = K_{it}/Q_{it} \), Walras’ law implies

\[
\sum_i (s_i \pi_i u_{it} - g_{it-1} v_i) = 0
\]

where \( g_{it} \) is the growth rate of the capital stock, assuming for the moment a rate of depreciation of zero. Equation 3 sums the financial surplus over all \( i \) firms, where the financial surplus is the difference between the firm’s savings out of profits and its planned investment, normalized by the capital stock. It is the role of the financial system to channel funds from firms with positive to those with negative financial surpluses.

\[\text{Axtell (1999)}\] shows how agents combine to form firms, and this analysis can apply to both firms and traders in this model.
2.1 A two-sector example

Consider the solution to the model when the number of firms is \( n = 2 \). Suppressing the time subscript, the equivalents of equation 1 for a two-sector model can be expressed as

\[
x_1 = \theta [(1 - s_1 \pi_1)x_1 + (1 - s_2 \pi_2)x_2 + gK_1 + gK_2] \tag{4}
\]

\[
x_2 = (1 - \theta) [(1 - s_1 \pi_1)x_1 + (1 - s_2 \pi_2)x_2 + gK_1 + gK_2] \tag{5}
\]

Summing these two equations gives the savings-investment balance for the economy. Normalizing by \( K_1 \) and replacing the second equation by the savings investment balance, the model can be re-expressed as

\[
\frac{u_1}{v_1} = \theta \left\{ (1 - s_1 \pi_1) \frac{u_1}{v_1} + g_1 + \left[ (1 - s_2 \pi_2) \frac{u_2}{v_2} + g_2 \right] \frac{K_2}{K_1} \right\}
\]

\[s_1 \pi_1 \frac{u_1}{v_1} + s_2 \pi_2 \frac{u_2}{v_2} \frac{K_2}{K_1} = g_1 + g_2 \frac{K_2}{K_1}\]

Figure 1 shows the solution to the two-sector model when the savings rates are given and the first firm’s financial surplus is positive. In the third quadrant, total (normalized) investment, the right-hand side of the second equation above, is expressed as a function of last period’s level of capacity utilization in firm 1, \( u_{1t-1} \). Total investment is then reflected on the positive ordinate in the second quadrant to balance total (normalized) savings as a function of this period’s level of capacity utilization in firm 1, \( u_{1t} \). The firm shows a surplus since at \( u_{1t} \), its savings, \( s \), measured on the positive ordinate of the first quadrant, is greater than \( g_1 \) as reflected from the negative ordinate through the second quadrant.

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2. It is important to note that figures 1 and 2 provide a strictly transitory snapshot of the two firms at hand: surplus firms can very easily become deficit firms (and vice versa) between successive iterations of the model.
With only two firms, macroeconomic equilibrium implies that the second firm must be in deficit. Figure 2 shows the solution to the two-sector model with the second firm's financial surplus less than zero. The firm shows a deficit since at $u_{2t}$, its savings, $s$, measured on the positive ordinate of the first quadrant, is less than $g_2K_2/K_1$ as reflected from the negative ordinate through the second quadrant.

![Figure 2: Deficit firm](image)

At its current rate of capacity utilization, the firm in figure 1 is saving more than it plans to invest. This firm can self-finance its desired investment, and therefore faces no microeconomic financial constraint when executing its investment plans. The same cannot be said for the deficit firm in figure 2 which, at its current rate of capacity utilization, is attempting to invest more than it saves. This firm must borrow from the surplus firm in order to execute its investment plans. It is the role of traders in the financial sector to channel these funds from surplus to deficit firms.

In the simple two-sector example of figures 1 and 2 there is already an implicit financial sector linking the two real-side firms. In the long tradition of macro modeling, emphasis on the aggregate balance between savings and investment undermines the careful interrogation of this link. The flow goes on behind the scenes and macroeconomic balance comes about without attention to the specific mechanisms that are required. A central hypothesis of this paper is that omitting an explicit analysis of financial linkages creates an upwardly biased estimate of the growth potential of the system.

Even in the two-sector model depicted above, it is evident that something could easily go wrong with the flow of funds between the two firms. First, the deficit firm in figure 2 may not be able to borrow at all if it is unable to find a conduit or trader to intermediate. Agent-based models emphasize the locality of this search and so a first modification of the simple two-firm example is that the borrowing firm must be able to find a trader in its locale to channel funds from the lending firm. Only if loans are available and meet or exceed the borrower’s deficit, can the latter invest.

A second problem lies in the intermediation itself. Despite the existence of a surplus of loanable funds, there is no guarantee that a trader might not block the flow, effectively preventing financing from finding its way to the deficit firm. Since traders’ profits depend upon facilitating the flow, it may seem natural to assume that they will find an efficient way to channel resources from lender to borrower. If, however,
circumstances lead the trader to believe that the deficit firm will be unable to repay the loan, the trader may well defer, effectively preventing the investment that would otherwise take place. In reality traders, quite rationally, can and often do throw “sand in the gears”.

Observe that were the trader to block the flow from the first to the second firm in the example above, the level of activity at which savings comes into balance with investment depends only on the level of investment by the first firm. To the extent that intermediation is incomplete, part of the ex ante surplus of the lending firms simply evaporates, investment becomes financially constrained, and the system cycles down to an equilibrium in which aggregate savings is equal to aggregate investment at a lower overall level of economic activity. All this is brought about by the reluctance of the trader to serve as a conduit of funds.

2.2 Full capacity utilization and aggregate demand sharing

It is well known that structuralist models are quite capable of producing capacity utilization rates that exceed one and usually a judicious choice of parameters is required for the model to behave properly in this regard. In the multisectoral model here it is appropriate to ask what ensures that rates of capacity utilization do not exceed one.

In a one-sector model excess capacity utilization requires an adjusting variable to balance savings and investment. In multi-agent systems, however, a more complex set of problems present themselves. It is entirely possible, for example, that one firm reaches a capacity constraint while others remain slack. There should be some mechanism in the model that reallocates demand from firms with \( u = 1 \) to those with spare capacity. Then, only when all firms simultaneously encounter the capacity constraint, must some other variable adjust. The “judicious choice of parameters” mentioned above is thereby unnecessary until the economy as a whole is overheated.

Here the re-allocative mechanism is simply to adjust the demand shares in equation 2 above so that the demand that would have exceeded the capacity of the first firm, spills over to a second. Of course if no firm reaches full capacity utilization in a time period, demand shares need not be adjusted. If, however, some firm reaches full capacity, the share of a randomly selected firm rises to absorb the overflow. If the overflow causes the capacity utilization of the selected firm to rise above one, then its overflow is allocated to another randomly selected firm and so on. The mechanism halts when all firms are either at or below full capacity utilization. Using carets to denote diagonal matrices, the multi-sector analogue of equations 4 and 5 can be expressed compactly. With \( \mathbf{x} \) as an \( n \)-element column vector of firm outputs

\[
\mathbf{x} = \mathbf{\hat{Q}} \mathbf{u}
\]

the system can be written

\[
\mathbf{\hat{Q}} \mathbf{u} = \hat{\theta} \left[ \mathbf{D} \mathbf{\hat{Q}} \mathbf{u} + \mathbf{gK}_{t-1} \right]
\]

where

\[
\mathbf{u} = \begin{bmatrix} u \\ 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 1 - s_1 \pi_1 & 1 - s_2 \pi_2 & \cdots & 1 - s_n \pi_n \\ 1 - s_1 \pi_1 & 1 - s_2 \pi_2 & \cdots & 1 - s_n \pi_n \end{bmatrix}, \quad \mathbf{gK} = \begin{bmatrix} g_1 K_1 & g_2 K_2 & \cdots & g_n K_n \\ g_1 K_1 & g_2 K_2 & \cdots & g_n K_n \end{bmatrix}
\]

Since both \( \hat{\theta} \) and \( \mathbf{\hat{Q}} \) are diagonal and therefore symmetric the system can be solved for \( \mathbf{u} \)

\[
\mathbf{u} = \hat{\theta} \left[ \mathbf{\hat{Q}}^{-1} \mathbf{D} \mathbf{\hat{Q}} \mathbf{u} + \mathbf{\hat{Q}}^{-1} \mathbf{gK}_{t-1} \right]
\] (6)

The computational model solves the vector equation by way of the Gauss-Seidel method. At each iteration, the vector of \( \hat{\theta} \)'s is updated to reflect the firm’s new share of total aggregate demand that results from either its having encountered the full capacity constraint or having to satisfy spillover demand from a firm that has reached full capacity. The order in which firms are allocated spillover demand is random, so that no particular firm benefits from the procedure. Note that there is no implicit optimization of output here; the program simply looks for a basic feasible solution to a simultaneous set of demand equations under the constraint that no level of capacity utilization can exceed one. Once it finds a basic feasible solution, the Gauss-Seidel halts.
3 The financial sector

Network analysis is an increasingly common feature of economic and financial theory (see, for example, Iori et al. [2008] and the references therein). It is already well understood that the precise architecture of a network can affect its vulnerability to both external shocks (Barabási and Albert [1999]) and epidemics (Barthelemy et al. [2005]), which may be thought to be analogous to financial manias and panics.

The financial sector in this paper is organized into a financial network, which consist of nodes or vertices (individual traders) connected by one or more edges, which represent financial relationships between traders. As specified in more detail below, the links are conduits of borrowing, lending and trust in the formation of expectations about future market conditions. Financial agents are therefore said to be coupled by the links and the strength of the coupling is an important parameter of the computational model.

How networks propagate shocks depends on how they are constructed. In random networks, edges connecting vertices are generated randomly (Erdos and Renyi [1950]). The degree distribution, or frequency distribution of edges for each node, has finite mean and variance. Scale-free networks have a power-law degree distribution of link, which means that a few nodes have a number of edges, while most nodes have a small number. A network in which 20 percent of the nodes are connected by 80 percent of the edges is a Pareto-Zipf distribution or power-law degree distribution (Barabási and Albert [1999]). The network is scale free in that a power law applies to any segment of the distribution no matter how finely subdivided. Break up a normally distributed data set with mean of $\mu$ and standard deviation of $\sigma$ into smaller subunits. Each division will generally have a different standard deviation that will not be equal to $\sigma$, which is itself associated with only one of the scales on which the data is observed. In scale-free distributions no matter how finely the data is divided, the same distribution is in principle observed.

The key parameter of the distribution is the scaling parameter $\gamma$ in the probability distribution for degree, $d$, $zp(d) = d^{-\gamma}$, where $z$ is a calibration constant and is, ideally, scale invariant. The most notable characteristic of a scale-free network is the relative frequency of vertices with a degree that greatly exceeds the average.

In a random network, the spread of shocks requires that a certain minimum number of vertices are impacted, whereas in a scale-free network, this threshold is essentially zero (Iori et al. [2008]). The distinction between random and scale-free networks underlies models of bubbles and crashes such as in Sornette (2003). There, a specific geometry of nodes and edges, known as a hierarchical diamond lattice (HDL), produces a power-law degree distribution of links and is assumed to reflect the concentration of financial markets.

The financial networks of the model of this paper can be characterized by a binary adjacency matrix $A = A[a_{ij}]$ with $a_{ij} = 1$ indicating a network connection between traders $i$ and $j$. In non-directed graphs the adjacency matrix is symmetric, $a_{ij} = a_{ji}$, $\forall i, j$. The degree, $d_i$, of the $i$th vertex (trader) in the financial network is

$$d_i = \sum_{j \in J} a_{ij}$$

where $J$ is the set of all traders in the financial sector other than $i$. If the behavior of traders is based, in part, on observation of the behavior of other traders to whom they are coupled, then the degree of a vertex can be thought of as a measure of the influence that any particular trader’s actions will exert on other traders in the financial sector.

The degree distribution $p(d)$ will depend on the specific architecture of the network. Following Erdos and Renyi (1950) a random network is constructed by associating traders with one another randomly. A second mechanism, due to Barabási and Albert (1999), employs preferential attachment. Here, the likelihood that an additional trader created will be coupled with an existing trader varies directly with the degree of the potential partner. For example, suppose that trader $A$ is already linked to both traders $B$ and $C$, each of whom are connected only to trader $A$. There are thus four network connections in total. The probability that a new trader, $D$, will link to trader $A$ is $2/4 = 1/2$, whereas the probability that $D$ will link to $B$ (or alternatively, to $C$) is $1/4$.

A popular network textbook, Lewis (2009), claims that “scale free networks are are extremely nonrandom (emphasis in the original).” A better characterization, perhaps, is that scale free networks are extremely “non-Gaussian”.

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Preferential attachment provides a self-reinforcing mechanism according to which the most linked vertices in a network, those with the largest existing number of edges, are more likely to form additional links. Observe that preferential attachment is a mechanism that characterizes the growth of non-random networks. It is a simplified model of how networks come into existence rather than a description of existing networks, as in Iori et al. (2008), or how established networks change or reconfigure themselves in response to an external shock. Preferential attachment is nothing more than a simple set of rules that always generates a scale-free network. It is arguably a simpler, more realistic and less arbitrary representation of financial markets than the HDL in Sornette (2003). By generating networks both randomly and through preferential attachment, whether the latter makes the financial system more crisis prone can be tested.\(^4\)

The strength of a vertex is defined as
\[
\sigma_i = \sum_{j \in J} w_{ij}
\]
where \(w_{ij}\) is the weight attached to the edge between \(i\) and \(j\), and \(J' = \{ j \neq i : a_{ij} = 1 \}\). The weight is defined here by \(\omega_i = K_i / \sum_i K_i\), the share of real capital accumulation that trader \(i\) has financed through loans.\(^5\) The latter is a measure of the importance of trader \(i\) for the real sector. The strength of node \(i\) is
\[
\sigma_i = \sum_{j \in J} \omega_i a_{ij} = \omega_i \sum_{j \in J} a_{ij} = \frac{K_i}{\sum K_i} d_i
\]
The strength of a vertex so defined provides a second measure of the influence that any individual trader \(i\)'s actions will have on the behavior of other traders in the financial sector. This second measure is sensitive to both the trader’s degree, \(d_i\) and the share of the total capital stock that the trader has financed, \(\omega_i\), which is generated endogenously by the trader’s lending behavior in the course of economic activity.

This second weighted degree distribution, \(p(\sigma)\), produces a more extreme measure of the connectedness of the financial sector. Moreover, in models without associated real sectors, no such weighting scheme naturally suggests itself. Suppose that \(p(\sigma)\) involves a further departure from the Gaussian architecture, and that the proclivity of the economy to experience financial crisis increases with its departure from the Gaussian. It then follows that there are important real-sector foundations of financial crises, foundations not adequately accounted for in the stand-alone models of the financial sector.

### 3.1 The origin of preferential attachment

The skeptical reader may reasonably ask why preferential attachment deserves the emphasis it is given in the model and, moreover, how such a structure corresponds to the financial architecture of an actual economy. In stand-alone financial models arbitrary network geometry is invoked to produce a power-law degree distribution of network links.\(^6\) Preferential attachment generates the same degree distribution but is arguably more realistic. The assumption here is that concentration of the capital stock in large enterprises in the past corresponded to a parallel concentration of financial institutions.\(^7\) Thus, the preferentially attached financial network is the result of the evolution of concentration in the economy broadly and is understood as the product of past real-financial interactions.

### 3.2 Intermediation in Keynesian models

Traders’ ability to lend to deficit firms is based on the availability of deposits made by surplus firms. At the beginning of simulated time traders have an initial endowment of assets. During each subsequent period,

\(^4\)Note that preferential attachment is a simplification that comes at the cost of precluding clustering. This is ruled out since no existing trader can add a link to some other existing trader that has already attached itself to the network.

\(^5\)Observe that in creating these loans, the trader bears the risk of default, since the trader assumes ownership of the debt-leveraged assets operated by deficit firms. In contrast surplus firms borrow nothing and therefore both own and operate their production processes.

\(^6\)The reader is again referred to the use of the HDL in Sornette (2003).

\(^7\)See Rajan and Ramacharan (2009) for an account of how this may have happened in the U.S.
firms make deposits out of the savings from the profit income they generate with the nearest trader. Borrowed deposits do not accumulate however, since once borrowed they become illiquid.\(^8\) Surplus firms provide traders with loanable funds that enable traders to make loans to deficit firms but only those funds that have not already been lent, in previous periods, to others.

If, at any point in time, the client of the \(i\)th trader is a surplus firm, the change in loanable funds \(\Delta \ell\) of trader \(i\) is

\[ \Delta \ell_i = f_i \tag{10} \]

where \(f_i = S_i - g_i K_i\) is the current savings less planned investment for the next period of the trader’s client. Investment by deficit firms is then limited by the availability of loanable funds. Surplus firms, however, are never so limited: they can always execute their investment plans up to the full extent of their savings in the previous period.\(^9\)

The asymmetry of this last assumption deserves some comment. Deposits made by surplus firms are legally available to them. Consider time period \(t\). Deposits of savings from the period \(t - 1\) are assumed to be available to finance deficit firms. Borrowed deposits from the more distant periods, \(t - 2, t - 3, \ldots, t - n\) are not, since they have been used to finance investment by deficit firms in periods \(t - 1, t - 2, \ldots, t - n\). The model therefore assumes a binding savings constraint inherited from the previous periods and this gives the model some features that might not be immediately self-evident. It may appear that the model has lost its Keynesian character and has become a classical model in which prior savings determines current investment. This conclusion is not, however, fully warranted, since there remains an important role for autonomous investment. Investment in period \(t\) can exceed savings in period \(t - 1\), but only if surplus firms increase their planned investment.

To see why, consider a dilemma a surplus firm might seemingly encounter. Adequate deposits to cover the firm’s investment plans are present in the firm’s account, but when the firm attempts to use the financing to purchase capital goods, could it be blocked by the financial agent on the grounds the funds had already been loaned to a deficit firm? The answer is no: surplus firms are legally entitled to their deposits and so it is only under the extraordinary circumstances of a credit freeze that surplus firms would be barred from using its liquid savings for investment.\(^{10}\) The trader simply creates the money to reinstate the funds of the surplus firm. In this way there is a forced increase in the money supply, whether planned or not, by the monetary authorities. This forced increase in the money supply causes money to be partially endogenous in the simulation results discussed below.

Agent-based models thus make explicit how endogenous money comes about, since if deficit firms have already contracted to borrow the surplus, traders have no choice but to create the liquidity when surplus firms are ready to invest. This mechanism provides a multi-agent system micro-foundation for endogenous money. Although deficit firms can and do crowd out other deficit firms, they cannot crowd out surplus firms. In this way the model retains something of its Keynesian flavor, since it is animal spirits that ultimately allow aggregate investment in period \(t\) to exceed savings in period \(t - 1\).

Observe that without such monetary creation, the model is interconnected from period to period in a way that makes savings determine investment. If funds available for deficit firms were always just what is left over after surplus firms make their investment, a sequence of events would unfold in this case such that

\[ I_0 = S_0 \geq I_1 = S_1 \geq I_2 = S_2 \ldots \tag{11} \]

where \(I_0\) is the initial level of investment. Observe that at no point in this sequence can \(I_{t+1}\) exceed \(I_t\). At best, investment is constant over time and equal to its initial value.

At the same time, the capital stock is accumulating with each round of investment. With a fixed distribution of capital-output ratios, capacity utilization will therefore have to contract, reducing investment

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8 Any unborrowed deposits do, of course, accumulate and enhance the ability of deficit firms to invest in future periods.

9 Note that if a surplus firm elected to spend more than authorized by previous period savings, it would become a de facto deficit firm.

10 To deny surplus firms the use of their own savings is to announce catastrophic financial failure. In this worst-case scenario, a system-wide run on deposits would likely occur. No attempt to model such a financial melt-down is made in this paper, although it could be the result of the financial crises that do occur.
below its savings-constrained value, since the latter depends on the utilization rate. So long as previous savings is a binding constraint on investment in the subsequent period, the envelope of sequence [11] is at best a stationary state, since

\[
\frac{I_0}{K_0} = \frac{S_0}{K_0} \geq \frac{I_1}{K_1} = \frac{S_1}{K_1} \geq \frac{I_2}{K_2} = \frac{S_2}{K_2} \ldots
\]  

(12)

where each period’s savings-investment balance has been normalized by the contemporaneous capital stock. Note that if \( \hat{I} \) is not equal to \( \hat{K} \), the stationary state will not obtain. To see this note that the savings constrained sequence in equation [11] implies that the rate of growth of investment is at best zero. All a non-monetary Keynesian real economy can do is to reach a stationary state.

Observe that there can be no net capital accumulation with a fixed capital-output ratio along the envelope. This is because accumulation would reduce capacity utilization and thus investment from then on. In the stationary state, all investment is, by definition, necessary to maintain the capital stock at its current level. The envelope provides an upper bound for possible trajectories of the non-monetary model; any departure from past capacity utilization implies an interior trajectory for the model.

In a model with net capital accumulation, the rate of capacity utilization will remain constant over time if and only if investment grows at the same rate as the capital stock [Gibson, 2009]. This implies that at some time \( t, S_t \leq I_{t+1} \). It follows that in the steady state, both savings and investment are growing at the same rate and equal to the rate of monetary creation. Without new money creation, the economy is doomed to remain inside, and perhaps deeply so, the envelope of the sequence in equation [12]. Otherwise “bridging finance,” to use a term introduced by [Chick, 1983], must be introduced to allow investment to exceed previously accumulated savings.

Rather than acting as a binding constraint on aggregate investment, the sequential, savings-in-advance nature of the model instead imposes a particular structure on the behavior of traders. Deficit firms are dependent on traders to act as intermediaries and must obey the inequality

\[
I_{dt} \leq S_{dt-1} + f_{t-1}
\]  

(13)

where the subscript \( d \) refers to deficit firms and \( f_{t-1} \) is the financial surplus of the surplus firms in the period \( t - 1 \), or, \( S_{st-1} - I_{st-1} \). Surplus firms, on the other hand, are free to invest

\[
I_{st} \geq I_{st-1}
\]  

(14)

where the change in the supply of money is

\[
\Delta M \leq I_{st} - I_{st-1}
\]  

(15)

As discussed above, traders create money in the event that the financial surplus of the previous period has already been committed to the deficit firms. If deficit firms fail to use all the available financial surplus, then the required monetary easement will, of course, be reduced to

\[
\Delta M = I_{st} - I_{st-1} - f_{t-1}^* \]

where \( f_{t-1}^* \) is the underutilized financial surplus. In other words, for monetary expansion to take place, it is also necessary that investment demand on the part of deficit firms be sufficient to use more of the financial surplus in equation [13] than would be necessary to fund the additional investment surplus firms wish to make. If equation [13] holds as an equality then so too must the expression in equation [15]. The performance of the economy depends in large measure on the level of monetary injection that results from this process.

Observe that there would be no reason to distinguish surplus and deficit firms if credit or money were fully endogenous. Firms that lacked sufficient savings from the previous period would simply borrow for investment from bankers who are, in turn, able to create money. In that case the Keynesian nature of the economy is fully restored. Given a set of expectations that the economy will stay on a steady-growth path, nothing prevents the system from expanding without limit. Figure [3] shows two possible limiting cases. The
upper trajectory corresponds to full capacity utilization, with fully endogenous money. The lower trajectory is that just described, along which the economy is limited by previous savings.

The economy described by the lower path will eventually turn down. The reason: there may be impediments to investment on the part of any given firm that arise in the financial sector. Either a deficit firm is rejected by the trader or the deficit firm cannot locate a trader willing and able to lend in the local region of the grid. If investment by a deficit firm is blocked for either of these reasons, then total investment falls and with it available savings for the next period. Firms that would otherwise have been in surplus now are themselves in deficit if their investment plans are not scaled back to match their savings. With a sufficient contraction of demand, all firms can fall into deficit simultaneously and the result will be a sharp contraction in investment in the following period. This imparts a negative trend to GDP as seen in figure 3 since local frictions will overpower endogenous credit creation. Any real economy operates inside the shaded region of the figure.

The principal implication of this theoretical analysis is that Keynesian economies are more monetary than is often recognized. On one hand, the simplistic view that money is always and everywhere fully endogenous neglects the power of the financial sector to block the flow of finance from surplus to deficit firms. On the other, there is no net expansion in the economy without some monetary or credit creation. Central banks can and do yield to the temptation to increase available credit to keep the savings constraint from binding. Were they instead to switch to an anti-inflationary strategy, some firms would run short of funds when attempting to invest. Note that as modeled, the financial sector reflects the spirit of Kalecki’s principle of increasing risk, by making the execution of planned investment easier for surplus firms, which are investing their own capital, than for deficit firms that need to borrow in order to invest (Kalecki 1937).

There is no claim that a highly financially constrained real sector is a realistic account of actual economies. The framework is, however, a useful testbed to study crises. In standard structuralist models, crises never arise endogenously. These models can, of course, be altered to incorporate features that give rise to a crisis, but if crashes are infrequent, it becomes difficult to assess whether they arise from pure chance given the parameter settings in the model or from more systematic causes. It may be argued that the real sector here is too vulnerable to crises. By allowing for crises to arise with greater frequency than in real economies, however, the model produces outcomes that can be subjected to statistical analysis, as in the last section of the paper.

### 3.3 The trader’s forecast

Even if traders have liquidity and are able to lend, the question still remains as to whether they will be willing to provide the needed finance. In the model, the individual trader’s financing decision depends on...
a forecast, essentially whether trader sentiment is either bullish or bearish about the future. Following Gonçalves (2003), the forecast of each trader depends on a combination of two sources of information, or signals, plus random error. The first is an idiosyncratic private signal, defined as the capacity utilization of the trader’s client. If the capacity utilization of the firm associated with any given trader falls below a threshold level, that trader turns bearish and the private signal is set to -1. This is implemented in the model subject to a uniformly distributed random error, however, so that even if the trader’s firm is operating with significant excess capacity, the trader may still reject the bearish private signal and report a bullish forecast.

The second component of each trader’s forecast is the subjectively perceived social signal. This arises from the trader’s perceptions of the forecasts of other traders to whom the individual trader is linked in the financial networks discussed earlier. The social signal is included to capture Keynes’ well known “beauty contest” or noise trading. In an efficient market, asset prices should convey all available information, but with noise trading, the price can rise to reflect what traders believe other traders believe. Following Schleifer (2000) it is rational for even fully informed traders to follow noise so long as they can exit the market before a crash occurs. Traders are indexed \( i = 1, 2, \ldots, n \) and \( J' \) denotes the neighborhood of agents connected to agent \( i \). When agents are linked in a financial network, they exchange information. There are only two possible forecasts, \( \phi \), for the \( i \)th trader

\[
\phi = \begin{cases} 
-1 & \text{bear or pessimistic} \\
1 & \text{bull or optimistic} 
\end{cases}
\]

The signal extracted by trader \( i \) from the behavior of her neighbors is \( \sum_{i \in J'} \phi_i \). Note that an informed trader maximizes her return by having taken the “right” signal from her colleagues. The signal is the right signal if in fact her neighbors are representative of the trader agent set as a whole. In that case, asset prices will have moved in the same direction as the neighbors’ forecast. Were the trader to act on fundamentals alone, only the idiosyncratic private signal would matter. In fact, the trader’s forecast combines the idiosyncratic signal and the subjective social signal that arises from perceptions of other traders’ behavior. The forecast of trader \( j \) is thus determined as

\[
\phi_j = u_j - \bar{u} + K(\omega) \sum_{i \in J'} \phi_i + \epsilon
\]

A random error term allows traders to override their private and/or social signals and set a forecast based on intuition. Note that the implicit weight of the private signal in the calculation of \( \phi_j \) is one, so that \( K(\omega) \), the “coupling” parameter, is normalized by the value of the strength of the private signal.

The value of \( K(\omega) \) measures the degree of influence exerted by other traders on the forecast of trader \( j \). Observe that \( K \) is weighted by \( \omega \) because traders are not the same size: more weight is attached by any given trader to the forecast of large traders in the derivation of the social signal. As noted above, the weights are proportional to the values of the capital stock under the control of each trader.

The coupling parameter evolves as in standard “spin-glass” models. The coupling parameter varies subject to confirmation bias: if the share price, \( p \), moves in a direction consistent with the forecasts of the traders to whom trader \( j \) pays attention when forming the forecast, \( K \) rises and vice-versa. Each trader attaches increasing weight to what it perceives as the superior information possessed by other traders. Influence becomes self-reinforcing since an uptick in the share price accompanied by \( \sum_{i \in J'} \omega_i \phi_i > 0 \) initially will increase the coupling parameter, raising the likelihood that \( \phi_j > 0 \) and hence the likelihood that the share price will rise again, which will further increase the coupling parameter and so on. The path of the coupling parameter is then:

\[
K_{t+1} = K_t + \sum_{i \in J'} \omega_i \phi_i, \quad \text{for } t = 0, 1, 2, \ldots
\]

Without this random error term, the modeled economy would never recover from a crash: cascades would go on forever.

If the change in the share price and \( \sum_{i \in J'} \omega_i \phi_i \) are of opposite sign, the coupling parameter is reduced by the value of \( \sum_{i \in J'} \omega_i \phi_i \).
If this process continues, the social learning implicit in $K \neq 0$ can break down, giving way to herding behavior and (in the limit, when all traders are herding) cascades [Chamley 2004]. Finally, if traders were bullish last period, the probability is slightly greater that they will be bullish this period, everything else equal.

### 3.4 Preferential attachment and lending

If lending agents do not have the liquidity necessary to satisfy the demand for borrowing by deficit firms, they may call on their linked neighbors to ask for a loan. Linked traders who have sufficient funds can agree to loan the originating trader the balance to meet the demand for liquidity of the deficit firm. The depth of this lending relationship is limited to one ply. Observe that were there no such limitation, deficit firms would have access to the entire network, assuming it was connected and did not break down into sub-networks.

![Decision Tree](Image)

**Figure 4**: Financing investment of the deficit firm

Figure 4 describes the decision tree with a deficit firm petitioning a local trader for a loan. If the trader is bearish, that is the forecast is negative, then the loan is denied and the investment is blocked. If the trader is bullish, however, but short of liquidity, the trader asks his linked neighbors if they can make the funds available. Crucially, the loan does not depend on the forecast of the linked neighbors. The loan originator in this way bears all the risk, while counter-party surveillance of the originator by the linked neighbor is effectively nil. Observe that this is the point at which financial leverage comes into the model.

Again the structure of the network is hypothesized to be crucial to the crash propensity of the system. With preferential attachment, bullish traders with many linked neighbors will be more able to make loans, and so their ability to finance deficit firms will rise. A firm associated with one of these traders is much more likely over time to find finance for any deficit that might arise. As a result, firms with highly linked traders will tend to accumulate capital stock more easily and grow larger over time.

As a high-degree hub turns bearish, however, it blocks loans to its own client if at that instant, the client is in deficit. This, of course, reduces the level of effective demand in the system and to a greater degree since the firm’s capital stock will likely be large. As large firms reduce their investment demand, other surplus firms may well go into deficit. Preferential attachment concentrates this kind of demand shock to the system in the hands of a few traders. On the other hand, highly linked, bearish traders having refused loans to their own client’s then have more liquidity to pass along to their bullish linked neighbors. Sorting out the
net effect of these currents and counter currents on aggregate demand, share prices, and the possibility of a crash is the job of the simulation model below.

3.5 Asset prices

The share price is defined as an index of traders’ forecasts. If forecasts are bullish, the share price rises and vice-versa:

\[
\sum_{i \in J} \phi_{i,t-1} > 0 \text{ price rises}
\]
\[
\sum_{i \in H} \phi_{i,t-1} < 0 \text{ price falls}
\]

where \( J \) is the set of all traders. The equation for the share price is

\[
p_{t+1} = p_t + \psi \sum_{i \in J} \omega_i \phi_{i,t}
\]

where \( \psi \) is an arbitrary parameter determining the scale of the marginal sensitivity of the share price to variations in traders’ forecasts. If there is a balance between bulls and bears, the share price remains constant. If bulls outweigh bears \( p_{t+1} \) rises and vice versa. The share price is a random walk during “normal” times, but breaks out during organized bull or bear markets to produce a bubble and then either a crash or soft landing.

4 Simulations

4.1 Simulation design

As noted, agents are divided into two disjoint subsets, firms and traders. A firm is instantiated with randomly assigned capital stock, savings rate, direct labor coefficient, and parameters of an investment function that depends on both animal spirits and capacity utilization in the previous period. Traders are instantiated with a forecast, a coupling coefficient (which can change according to the spin-glass mechanism described above), and linked neighbors with a network structure that depends on whether the model is run with or without preferential attachment and share weights. Traders initially have randomly assigned sensitivities to the public and private signals they receive. They also have random opinion volatilities, effectively the error term that allows them to ignore the variably weighted combination of their private and public signals. Traders must also keep track of deposits and shares in the firms to which they make loans. Trades and production are tracked on a weekly basis.\(^{13}\)

4.1.1 Finding crashes

How the program finds a crash is highly stylized. Figure 5 shows the method for a sample case. Using historical series for the S&P 500, it was determined that a “typical” build and crash involved some 250 weeks in total. A conjoined set of triangles with dimensions as shown in the figure is then applied to every 250 week period in each run. If the pattern of price movements fits the triangle, the program records a crash and halts.\(^{14}\)

To see how this method works, consider the share price for a run at time \( t > 250 \).\(^{15}\) From figure 5 it can be seen that a “build” is nothing more than an increase in the share price from a period 200 weeks before the

---

\(^{13}\)The simulations are done in Netlogo.

\(^{14}\)Neftci (2008) provides a more sophisticated and comprehensive method to date crashes that could also be implemented. The method requires calibration however and thus could be adjusted to get approximately the same number of crashes as the vastly simpler procedure employed here.

\(^{15}\)The method does not find crashes in the first 250 weeks.
current price at period $t - 50$. Thereafter a decline of 50 percent in the share price must take place within the next 50 weeks for a crash to register.

This method is necessarily subjective since some observers might see crashes where the program does not. The limited horizon for the drawdown, however, distinguishes a crash from a correction. Note that the method does not necessarily find peaks or troughs and therefore cannot be used to quantify the size of a given drawdown. Finally, its criterion for a build is designed to rule out a series that declines for a long period and then accelerates its decline. Despite its subjectivity, the principal advantage of the triangular pattern recognition is consistency across runs, in that it finds crashes that roughly agree with the observed build-crash sequence in recent U.S. financial history.

4.1.2 Simulation settings

Figure 6 provides a typology of the different settings used in the simulations of the following section. The primary distinction is, of course, whether there exists a binding financial constraint, that is whether money is partially or fully endogenous. Thereafter, financial networks can be either random or structured by the growth process, subject to preferential attachment, and either weighted or unweighted. The figure indicates how the simulation settings were selected to isolate these essential aspects of real-financial interaction. The simulation results presented below are organized according to the structure in the bottom row of cells in the figure.

Note also that when money is not fully endogenous, it is possible for the real sector to severely contract. This is defined by aggregate capacity utilization falling to a value of 0.6. A contraction will occur if, for example, an initial shortage of liquidity available to deficit firms reduces aggregate investment and hence the quantity of savings available to fund investment in the following period. The reduction in investment will also reduce capacity utilization, producing private signals that may result in traders becoming bearish. If these developments make for a further decline in investment and if the process just described continues unchecked, the result will be a steep contraction of the real sector that inevitably triggers a financial crisis as all traders turn bearish. This kind of crisis, one that results from a full collapse of the economy is not counted as a crash in the tabulations below.
4.2 Simulation results

4.2.1 Preferential attachment and the degree distribution of traders

The simulation model is run with two initial settings that configure the architecture of the underlying grid of traders. The first is simply a random network in which the traders are spawned on the grid and then linked neighbors are randomly connected. The resulting degree distribution that shows the frequency of high- and low-degree traders is illustrated for a particular initialization in figure 7. In log-log space the distribution is not linear but concave. A power-law, in contrast, will produce a characteristically linear degree distribution. As shown in figure 8, which again pertains to a particular initialization, this is what emerges when network formation is subject to preferential attachment. A linear fit is used to estimate the critical exponent at approximately 1.68, a number typical of networks constructed in this way.

4.2.2 Financial crises

Tables 1, 2 and 3 show the patterns of financial crises observed over 10,800 simulations of 1,500 period each and under different assumptions about the monetary regime. The tables follow the structure of 6. Table 1 shows the frequency of financial crises when money is fully endogenous. Table 2 is for money is only partially endogenous, and table 3 shows crash frequency when money is fully exogenous. Each table distinguishes between the initial setup of the financial network, whether random or resulting from preferential attachment, and then whether the network is share weighted.

Tables 1, 2 and 3 also distinguish between two qualitatively different types of financial crises resulting from either herds, or from herding behavior that leads to a cascade. A herd is defined as a subset of traders who all take the same action (for example, selling financial assets) after some date. An individual trader is said to engage in herding behavior, meanwhile, if he/she buys an asset purely in
Figure 7: Degree distribution for a typical random network.

Figure 8: Degree distribution with preferential attachment.
response to the observation that other traders are buying. Herding by an individual agent involves acting purely in response to a social rather than private signal and as such there is no social learning. (Chamley 2004) Herds, moreover, create order in financial markets that normally thrive on disorder.

Not all herds are comprised of individuals engaging in herding behavior. A herd can result from individual traders coincidentally making the same forecast with reference to their private signals. As previously noted, this can happen in the model. A sharp contraction of the real sector may well cause the private signals of enough traders to turn negative so that a financial crisis occurs. When money is fully endogenous, however, no such private-signal crashes happen, since the buoyancy of the economy prevents private signals from simultaneously turning negative. The first row of table 1 confirms this. Even when money is partially endogenous, there are no private signal financial crises, as seen in the first row of table 2. Evidently the financial constraint is not sufficiently severe to provoke a financial crises resulting from the impaired aggregate performance of the real sector.

When money is exogenous and the financial constraint binds most tightly, private signal crises do indeed materialize. In the first row of table 3 most runs result in catastrophic collapse of the economy, associated with aggregate capacity utilization falling below 0.6. Here a subtle but important difference emerges: in random networks, intermediation is less efficient and so the modeled economy always collapses. With preferential attachment, however, the flow of finance is freer, the financial constraint does not bind as tightly, and so catastrophic collapse of the economy is less frequent.

Even in the absence of a sharp contraction of the real sector, social influence can result in the formation of herds by giving rise to herding behavior. This can occur if, in the presence of a sufficiently large coupling coefficient, the private signals of high-degree hubs turn negative. If the resulting orderliness in the market reaches a point where all traders engage in herding behavior, the outcome is an informational cascade (Bikhchandani et al., 1992). A cascade will precipitate a financial crisis independently of the aggregate performance of the real sector. The second row of table 1 shows that such social signal crises do occur, even when money is fully endogenous. When the initial financial network architecture is random, financial markets are crisis free. When the architecture is preferentially attached, however, financial crises occur in 2-3 percent of cases. The frequency of these crises seems to be little affected by whether non-random networks are weighted by capital shares.

The second row of table 2 shows that when money is partially endogenous, preferential attachment again has an important effect on the frequency of crises due to herding behavior, while the influence of share weights appears negligible. Of particular interest is the markedly higher frequency in table 2 as compared to table 1 of these social signal crises in networks formed by preferential attachment. This suggests that, given the “right” network architecture, the performance of the real sector influences the frequency with which social signal financial crises occur.

The second row of table 3 shows that, despite the frequency of private signal crises when money is exogenous, it is still possible for herding behavior to precipitate a financial crisis before a severe contraction occurs. Indeed, the frequency of such events as reported in table 3 is noticeably higher again than its frequency as reported in table 2. Financial crash frequencies are higher under more restrictive monetary regimes. In non-random financial networks, real sector performance influences the frequency of financial crises by raising the likelihood of herding behavior and cascades.

---

16 The reason for this is as follows: if there is no monetary easement and the economy is savings constrained, deficit firms would have to have access to all possible sources of financial surplus on the grid simply in order to maintain a steady-state. Any impediment to the flow of finance would cause investment to fall and the economy to contract. By definition, high-degree traders have access to many more sources of leverage from linked neighbors than their low-degree counterparts and thus are less likely to frustrate their deficit clients. In random networks there are many fewer of these high-degree hubs, so intermediation and hence real sector performance suffers.
Table 1: Results of simulations: Money fully endogenous

<table>
<thead>
<tr>
<th>Preferential Attachment?</th>
<th>Share weights?</th>
<th>Share weights?</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nature of crash</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private signal</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Public signal</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Total runs</td>
<td>900</td>
<td>900</td>
</tr>
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</table>

Source: Authors’ computations
*Crises per 100 runs of 1500 weeks (30 years).

Table 2: Results of simulations: Money partially endogenous

<table>
<thead>
<tr>
<th>Preferential attachment?</th>
<th>Share weights?</th>
<th>Share weights?</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nature of crash</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private signal</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Public signal</td>
<td>0</td>
<td>6.2</td>
</tr>
<tr>
<td>Total runs</td>
<td>900</td>
<td>900</td>
</tr>
</tbody>
</table>

Source: Authors’ computations
*Crashes per 100 runs of 1500 weeks (30 years).
Table 3: Results of simulations: Money exogenous

<table>
<thead>
<tr>
<th>Preferential attachment?</th>
<th>Share weights?</th>
<th>Share weights?</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
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<td>Yes</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>86.4</td>
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<tr>
<td>900</td>
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<td>900</td>
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<tr>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>13.6</td>
</tr>
<tr>
<td>900</td>
<td>900</td>
<td>20.2</td>
</tr>
</tbody>
</table>

*Source: Authors' computations

*Crashes per 100 runs of 1500 weeks (30 years).

4.2.3 Significance

The results in tables 1 to 3 suggest that preferential attachment affects the frequency of social signal financial crises, and that in the presence of preferential attachment, the frequency of these events is also affected by the degree to which money is endogenous and possibly also the presence of share weights. This subsection explores the crucial question of whether these observations are in fact statistically significant.

Some fifteen million observations were prepared for the study. To analyze the effect of preferential attachment, for example, it is possible to select a subset of runs with identical settings for the financial constraint and share weights, two statistically equivalent sub-populations. To one a treatment of preferential attachment is applied and the other serves as a control. The resulting frequency of crashes is then regressed in table 4 on the binary variable “preferential attachment”, and then compared to the inherent variability of crashes by way of a $t$-test in order to determine significance.

The first row of table 4, for example, shows results for two sub-populations of 3,100,208 that differ systematically only in whether the network structure is random or preferentially attached. The dependent variable is the frequency of social signal crashes. It is seen that preferential attachment is significant at the 1 percent level.\(^{17}\)

One of the most notable findings of the paper can be seen in the second column of the table in which the dependent variable GDP is shown to be positively related to the preferential attachment dummy. The network structure is evidently janus in nature, propelling the economy forward in times of high growth but increasing the vulnerability to crashes when growth becomes sluggish. The results in the first row of table 3 are highly unlikely to be the product of chance alone, with preferential attachment again significant at the 1 percent level.

The second row of table 4 records the effect of the financial constraint in table 3. Observe here the essential interaction between the real and financial sectors, since again the regressions only count social signal crashes. When the real economy slows, but does not collapse to a capacity utilization of less than 0.6, the financial sector responds in an amplified way, with the frequency of crashes increasing. Again, the table corroborates the findings in tables 1-3 showing that the effect is highly statistically significant.

The effect of the financial constraint on GDP is not strictly speaking a statistical test of a hypothesis given the built-in effects of the financial constraint on the real sector as discussed above. This raises the critical question of just how restrictive the financial constraint is on growth. One way to address this issue is

\(^{17}\)Observe that the exercise is not a regression in the ordinary sense of the word but rather a difference in means test.
### Table 4: Regression results

<table>
<thead>
<tr>
<th>Equation</th>
<th>Dep var: Crashes</th>
<th>Dep var: GDP</th>
<th>Dep var: Crashes</th>
<th>Dep var: GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Preferential attachment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.84e-05***</td>
<td>2.31e+01***</td>
<td>3.21e-05***</td>
<td>-1.12e+02***</td>
</tr>
<tr>
<td></td>
<td>(6.41e-06)</td>
<td>(7.56e-02)</td>
<td>(6.71e-06)</td>
<td>(7.58e-02)</td>
</tr>
<tr>
<td></td>
<td>Financial constraint</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.21e-05***</td>
<td>-1.12e+02***</td>
<td>1.85e-05***</td>
<td>3.15e+02***</td>
</tr>
<tr>
<td></td>
<td>(6.71e-06)</td>
<td>(7.58e-02)</td>
<td>(3.23e-06)</td>
<td>(6.11e-02)</td>
</tr>
<tr>
<td></td>
<td>Constant</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-4.30e-15***</td>
<td>2.40e+02***</td>
<td>1.85e-05***</td>
<td>3.15e+02***</td>
</tr>
<tr>
<td></td>
<td>(7.67e-17)</td>
<td>(5.12e-02)</td>
<td>(3.23e-06)</td>
<td>(6.11e-02)</td>
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<tr>
<td></td>
<td>$R^2$-adjusted</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.028</td>
<td>0.000</td>
<td>0.385</td>
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<td></td>
<td>$R^2$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.0000317</td>
<td>.0285</td>
<td>7.72e-06</td>
<td>3.85</td>
</tr>
<tr>
<td></td>
<td>Observations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3,100,208</td>
<td>3,100,208</td>
<td>3,242,229</td>
<td>3,242,229</td>
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<td></td>
<td>114</td>
<td>93,092</td>
<td>22.8</td>
<td>2,189,112</td>
</tr>
</tbody>
</table>

Source: Authors’ computations based model calculations.

Standard errors in parentheses. *** p < 0.01, ** p < 0.05, * p < 0.1

Notes: 1. The dependent variable is either crashes per 100 runs of 1500 weeks (30 years) or real GDP.

To investigate the effect of relaxing the financial constraint, so that money is partially endogenous. In table 5 savings is not a strictly binding constraint on investment and it should follow that the economy is less crash prone and GDP growth somewhat more robust. Observe that crash incidence goes down, but even with three million observations, the result is only significant at the ten percent level. This is an expectedly weak effect of a relatively weak change in the model. On the other hand, GDP rises vigorously and significantly.

Table 5 also reveals some compelling results regarding the impact of share weights on crash frequency and GDP. First, the impact on crashes, which appears weak in tables 1-3, is positive and statistically significant at the one percent level. Second, share weights have a negative and highly statistically significant impact on GDP. Since the treatment “share weights” is only applied to preferentially attached networks, both effects are conditional on preferential attachment. Share weights do indeed increase the crash propensity of preferentially attached networks, but reduce the salutary effect of preferential attachment on growth. This suggests the idea that financial institutions are “too big to fail” has a more subjective/social component than is usually acknowledged, and that such institutions are instead “too big to save”.

## 5 Conclusions

This paper strives to construct a simple model of real-financial interactions, and to reflect on the frequency of any financial crises observed in the model. It is evident from the results that the model has produced that preferential attachment gives rise to financial market structures that are more fragile or crisis prone, and that the performance of the real side contributes to the frequency with which such crises are observed. These results confirm the importance of network architecture for the fragility of financial systems, and suggest that there are important real foundations of financial crisis. Just as [Kregel, 1985](#) argued that Keynesian real-side
Table 5: Regression results-continued

<table>
<thead>
<tr>
<th>Equation</th>
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<th>8</th>
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</thead>
<tbody>
<tr>
<td>Dep var: Crashes</td>
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<td>2.95e+02***</td>
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<tr>
<td>(7.94e-06)</td>
<td>(8.82e-02)</td>
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<tr>
<td>Dep var: GDP</td>
<td>-1.14e+01***</td>
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<tr>
<td>(6.91e-02)</td>
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<tr>
<td>Endogenous money</td>
<td>-1.15e-05</td>
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<tr>
<td>(6.80e-06)</td>
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<tr>
<td>Share weights</td>
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<tr>
<td>(7.94e-06)</td>
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<tr>
<td>Constant</td>
<td>4.17e-05***</td>
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<tr>
<td>(4.84e-06)</td>
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<td>$R^2$-adjusted</td>
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<td>8.73e-07</td>
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<td>.2145989</td>
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<td>5.83e-06</td>
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<td>Observations</td>
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</tr>
<tr>
<td>3,419,990</td>
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<td>$F$-stat</td>
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<td>16,695</td>
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<tr>
<td>Source: Authors’ computations based model calculations. Standard errors in parentheses. *** p &lt; 0.01, ** p &lt; 0.05, * p &lt; 0.1</td>
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<tr>
<td>Notes: 1. The dependent variable is either crashes per 100 runs of 1500 weeks (30 years) or real GDP.</td>
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</table>

models with no monetary and financial sectors were akin to “Hamlet without the prince”, so, too, it seems that stand-alone financial models that neglect the real side are incomplete. Ultimately, it seems that neither stand-alone real sector nor stand-alone financial sector models are suitable instruments.

6 Appendix: Pseudo code

The program can be expressed as:

1. Initialize data structures and runtime options
2. Set key parameters
   (a) Set share weights–boolean
   (b) Set preferential attachment–boolean
   (c) Set financial constraint–boolean
   (d) Set run years–30 × 50 weeks
3. Set up and initialize network
4. Reassign traders such that each firm has at least one trader
5. Set shareholders as count traders for each firm
6. Initialize surplus of each firm based on randomly assigned parameters
7. Run main
8. If financial constraint = FALSE: set invest = TRUE for all firms
9. If financial constraint = TRUE:
   (a) Ask traders: make forecast based on last period’s private and public signals
   (b) Ask firms: if surplus > 0 set invest = TRUE
   (c) Ask firms: if surplus < 0 ask one of traders if loanable funds |surplus|
      i. If yes: set invest = TRUE
      ii. If no: ask linked neighbor: if loanable funds > |surplus|
         A. if yes: set invest = TRUE
         B. if no: set invest = FALSE
         C. update denied-loan counter

10. Run Gauss Seidel [sum of investment of firms with invest = TRUE]
    (a) Set demand shares of firms
    (b) Set capacity utilization of firms
    (c) Set savings of firms
    (d) Set planned investment
    (e) Set surpluses of firms
    (f) Set loanable funds = surpluses of surplus firms

11. If share-weight = TRUE
    (a) Re-weight links by accumulated capital stocks

12. Stop for crash
13. Stop for year limit
14. Stop for capacity utilization lower limit (0.6)
15. Return to main
16. Process output

References


