ABSTRACT
This paper studies the consequences of debt policies on the spatial distribution of output in a two-countries set up. It departs from the usual set up of tax competition by relaxing the assumption of balanced budget. Expected taxation other than current tax levels motivates migration. Starting from an initial spatial configuration, be it core-periphery or symmetric equilibrium, the analysis identifies the critical thresholds of divergences or convergence of debt-ratios which break the initial configuration. The paper also shows that a high-debt country is a weaker player in tax competition and that tax harmonisation may not necessarily reduce migration flows.

J.E.L. classification: F15, F42, F22
Key words: Public Debt, Economic Geography, Factors Mobility.
Introduction.

The scope of this paper is to study the consequences of debt policies on the geographical distribution of economic activity. Changes to the debt policy imply changes to the inter-temporal tax profile faced by any mobile taxpayer. These changes may induce mobile taxpayers to migrate. The migration of taxpayers then results in a new geographical distribution of economic activity. Differently from the tax competition literature, in this paper governments are not in balanced budget. Therefore, migration is driven by the inter-temporal aspect of taxation rather by the current tax rates differentials.

The paper starts with the study of the effects of a pure inter-temporal reallocation of the tax burden. The inter-temporal reallocation of taxation is operated by one country only and, therefore, it generates a temporary divergence of debt ratios between countries. It is found that when factors are internationally mobile debt is “quasi-neutral” in the sense that it is neutral on the interest rate but it is not neutral on the current account. Further, public debt is also not neutral on the spatial distribution of output because causes a permanent international reallocation of factors away from the temporarily indebted country.

The paper then considers the same inter-temporal tax reallocation but this time in a set up where there are agglomeration and dispersion forces. It is found that the temporary divergence of debt ratios may generate a spatial fluctuation of output or agglomeration of output away from the temporarily indebted country. Perhaps more surprisingly, the analysis reveals that a policy of convergence of debt ratios also generates migration away from the debt-reducing country. This migration may generate a spatial cycle or agglomeration. Interestingly, in the numerical simulation it appears that a policy of debt convergence is more likely to cause agglomeration than a policy of temporary debt divergence. Finally, the paper points at the relationship
between debt policies and tax competition. It shows that the ability of a country to engage in tax competition is weakened by the presence of a debt overhang (be it permanent or temporary). Conversely, a policy of debt convergence strengthens the ability of the debt-reducing country to engage in tax competition in the long run but makes the debt-reducing country a weaker player in the short run. These results seem imply that large (core) countries with high debt should be averse to tax competition while small countries with small debt should be averse to tax harmonisation.

**Relationship to the literature**

This paper relates closely to the literature on taxation policies and economic geography. This literature is very recent and, as Baldwin et al. (2003) point out in their concluding chapter, a lot of stimulating issues remain yet to be explored.

Perhaps the main feature of new economic geography models is that the dynamics of the spatial distribution of economic activity is very sensitive to changes in the value of parameters such as trade costs and migration costs (Krugman, 1991a, 1991b). This feature has generated new research that revisited tax competition utilising this kind of models. Quite a few interesting results emerge from this research effort (see Baldwin et al. 2003 for a comprehensive and insightful treatment). Baldwin and Krugman (2000) show that that economic integration needs not to bring about a race to the bottom of tax rates. They restrict the analysis to a particular spatial configuration, the core-periphery situation, where the larger country has already attracted the core of industrial output. In such situation they show that the core country can increase the tax rate up until the point where firms are indifferent between staying in the core and moving to the periphery where sales are smaller but the tax rate is also smaller. This shows that agglomeration is taxable. This taxation strategy
recalls the concept of limit pricing and it is named by the authors “limit taxing”. In contrast with traditional results of tax competition they show that, since agglomeration rents are concave, economic integration should result firsts in an increase of the tax gap but then a decline as trade costs further decline. Ludema and Wooton (2000) also show that economic integration may result in a decline of tax competition rather than an increase but for a different reason. They restrict the analysis to a particular initial spatial configuration known as the symmetric equilibrium and study how this equilibrium shifts in response to tax policies. The government taxes only the mobile factor and redistributes tax revenues to the immobile factor. The paper shows clearly the three effects that an increase in taxation has on the welfare of the immobile factor. The first two effects (redistribution and the loss of tax base) are also present in the traditional literature. The novel result is in the price index effect. When the mobile factor flees the country the price index increases in that country and this is welfare reducing for the immobile factor. But if trade costs are low, the price index effect is small. Since the benefits from attracting the mobile factor decline with a reduction of trade costs, economic integration may actually result in a fall of the intensity of tax competition rather than an increase. Kind et al. (2000) show that economic integration increases tax competition. They analyse tax competition between two countries trying to attract mobile capital. Governments levy a source tax on the mobile factor (capital) and a lump sum tax on the immobile factor. The paper shows that in the core-periphery case the country hosting the core could gain by levying a positive source-tax on capital. This result recalls again the concept that agglomeration is taxable because, other things equal, revenues are higher in the core. The paper also finds that, if countries are symmetric, they will both subsidise the mobile factor in the attempt to attract the core. Ottaviano and van Ypersele (2002)
study the efficiency of tax competition. They consider asymmetric size countries and study the efficiency of the spatial distribution of economic activity in the market outcome, and the tax competition outcome. They show that when trade costs are low the market outcome and the tax competition outcome are equally efficient, for intermediate trade costs the free market outcome dominates the tax-competitive one, and for high trade costs tax competition dominates the free market outcome. The first two situations are of agglomeration. Thus, unless increasing returns industries are fully agglomerated, tax competition is superior to the market outcome. Otherwise, only tax harmonisation can improve on the market outcome. Anderson and Forslid (2003) consider a model where amenities enter the utility function. Since amenities are financed by taxation, a shift of taxation from the mobile to the immobile factor tends to stabilise the symmetric equilibrium because it stabilises the international distribution of amenities. Further, this stabilising effect tends to reduce the destabilising effect of a reduction in trade costs. They also show that, other things equal, a symmetric increase in taxation destabilise the symmetric equilibrium.

I use a model in the same spirit of those utilised in the literature reviewed above but that differs in one important assumption. The papers reviewed above as well as most of the tax competition literature maintain the assumption of balanced budget throughout the analysis. In this paper I remove this assumption and consider the effect of the inter-temporal allocation of taxation on the migration of factors.

While the balanced budget assumption may be convenient, the reality, especially in Europe, clearly calls for an extension of the analysis that include deficits and debts. The ability to engage in tax competition and indeed the ability to use the tax instrument depend importantly on the country’s debt overhang, especially so when the country has committed to achieve targets on debt and deficit. The present paper
then departs from the pure tax competition game to study the consequences of the
debt policies. The focus shifts from current tax rates to the present value of future
taxation. The debt policies considered in this paper are aimed at reaching targets in
terms of debt and deficit ratios. In this paper, governments do not use debt policies to
influence the location of mobile factors but such policies do have consequences on
expected taxation and thereby on the migration decisions of the mobile factors.¹

The link between debt and migration was first recognised by David Ricardo in
his famous “Principles” but remains up to date somewhat under researched. To my
knowledge, only some of the literature on local public finance has addressed the issue.
Daly (1969) and Oates (1972) demonstrate that, if individuals have perfect foresight,
and if taxation is in the form of a property tax, the migration of tax payers leaves the
net wealth unchanged for both migrants and non-migrants. In their models, an inter-
temporal tax reallocation does not result in migration of factors. This neutrality result
is revisited by Wellisch and Richter (1995) who show that the neutrality result does
not hold if local taxes are residence-base taxes on mobile individuals. In this more
general situation an inter-temporal tax reallocation has wealth effects. They argue that
the migration distortions that result from different debt policies provide a strong
efficiency argument in favour of inter-regional harmonisation of debt policies. These
papers consider the link between debt and migration but their focus is different from
the one in the present paper. Their objective is to study the incidence of different
modes of financing a local public infrastructure (Daly, 1969; Oates, 1972) or to study
the debt policy that maximises the welfare of local residents (Wellisch and Richter,
1995); their set up is of a small-region partial equilibrium model. The present paper
takes a different focus. It studies the consequences of debt policies on spatial output

¹ This paper refrains from modelling and studying “debt competition” as it seems rather unrealistic.
fluctuations and on industrial agglomeration in a two-country general equilibrium model and in the presence of agglomeration and dispersion forces.

1. The Model.

The structure of the model follows closely the well known original structure of the core-periphery model but augments it with the government sector and with forward-looking workers.\(^2\) The reason for utilising this model is that it has the catastrophic dynamics that we want to retain in our analysis. The world is composed of two countries (1 and 2) and is populated by infinitely lived individuals that appeared on earth at time \(z = 0\) by fiat people. The world population is normalised to be 1 and is composed of two types of individuals who constitutes the only two existing factors of production: skilled and unskilled labour. These two factors of production are available in given quantities: there are \(\mu\) skilled workers and \(1-\mu\) unskilled. Unskilled labour is assumed to be internationally immobile and, for the sake of symmetry, we assume that each country hosts \((1-\mu)/2\) unskilled workers. Skilled labour may migrate.\(^3\) The fraction of skilled labour in country \(i\) at any time \(z\) is denoted by \(\lambda_i\), obviously, \(\lambda_i = 1 - \lambda_{1-i}\). There are two goods being produced: a homogenous good \(A\) and a horizontally differentiated good \(M\). Good \(A\) is produced by use of unskilled labour only. Its production technology requires one unit of unskilled labour input for one unit of output. Following a consolidated tradition in this literature, the homogenous good is traded at zero cost while the differentiated

\(^2\) There are a number of variants to the structure of the core-periphery model. These variants concern in particular the expectation regime (Baldwin, 2001), the assumption of factor specificity (Ottaviano 2001, and Ottaviano and Forslid 2003), the assumptions of inter-sectoral and international mobility of labour (Ottaviano, 1999). Utilising any of these variants would not affect the results.

\(^3\) The assumption that skilled labour is more internationally mobile than unskilled labour is one of pure convenience but it corresponds to the empirical evidence on migration flows. See, for instance, Shields and Shields (1989).
commodity is traded internationally at an iceberg type of trade costs: for each unit of M shipped only a fraction \( \psi \in (0,1] \) arrives at its destination, free trade occurs when \( \psi = 1 \). We denote with \( w_{Ai} \) and \( w_{Mi} \) the nominal wages in the two sectors in country \( i \). Given the utility function specified below, both countries produce the homogeneous good as long as \( \mu < 1/2 \). We assume this inequality throughout the paper so that, thanks to the CRS technology and free trade in the homogeneous good, the price of A and the agricultural wage \( w_{Ai} \) can both be normalised to be 1 and serve as the numéraire. Commodity M is produced by use of skilled labour only: the input of skilled labour for \( q \) units of output of M is \( F+\beta q \). The constant \( \sigma \) represents the elasticity of demand and the elasticity of substitution among varieties. After the normalisations \( F=1/\sigma \) and \( \beta=\sigma/(\sigma-1) \), the output per firm equals 1 and the profit-maximising price for sales at home and abroad are \( p_H=w_{Mi} \) and \( p_J=w_{Mi}/\psi \) respectively. The number of varieties of M produced in each country \( (n_1 \text{ and } n_2) \) is endogenous and, after the normalisations, we have: \( n_i = \lambda_i \).

Individuals enjoy consumption according to the felicity function \( u(z) = \ln c(z) \), where \( c(z) = M^{\alpha} A^{1-\mu} \), and M is a CES aggregate with elasticity \( \sigma \) of the N varieties of M produced in the world. The natural log form of the felicity function is assumed for simplicity. Given the subjective rate of time preference \( \rho \), at any time \( z \) each individual maximizes \( \int_{s=0}^{z} [\ln c(s)] e^{-\rho(s-z)} ds \) subject to the flow budget constraint and to the transversality condition imposed on the state variable. The following system characterizes the solution to this problem:

\[
\begin{align*}
\dot{c}_i &= [r(z) - \rho]c_i(z), & i=1,2; \quad s = A,M \quad (1) \\
\dot{h}_i &= r(z)h_i(z) + d_i(z) - c_i(z), & i=1,2; \quad s = A,M \quad (2)
\end{align*}
\]
where $h_{si}(z)$ is the stock of financial assets accumulated by an individual in $i$, $e_{si}(z)$ is individual expenditure, and $d_{si}(z) = w_{si}(z) - t_{si}(z)$ is disposable income given by the wage rate minus per-capita taxation.

Governments are assumed to be identical in all respect except for the timing of taxation. Government expenditure is assumed to be a constant proportion of gross domestic product. Since this proportion does not play any role it is convenient to save notation by assuming it to be zero in both countries at any time.\(^4\) It is assumed that government 2 keeps the debt to a constant level (zero for simplicity). Since government expenditure is also zero, this results in a zero tax rate for all individuals in country 2. Government 1, instead, limits itself to be solvent at any time. Its tax function is:

$$T_1(z) = \bar{T} + \theta(z), \quad (3)$$

where $\bar{T}$ is a constant and $\theta(z)$ is a function whose specification is chosen by Government 1 to suit the desired intertemporal reallocation of taxation. Indeed, the only difference between the two governments’ tax policy is in that government 1 “plays” with $\theta(z)$ instead of assuring balanced budget at all times. Much of this paper studies the consequences of two types of policies: an inter-temporal tax-reallocation (which generates a temporary divergence of debt ratios) and a convergence of debt ratios starting from different but constant levels.

Taxation is distributed between the skilled and unskilled labour according to an exogenously determined equity criterion: a proportion $\Phi = \varphi \lambda \mu$ of total tax revenues is collected from skilled labour and the remaining $1 - \Phi$ is collected from unskilled labour. The parameter $\varphi$, which represents the equity criterion, can take any

\(^4\) The consequences of government expenditure on the location of output have been studied in Brülhart and Trionfetti (2003) in a similar model.
value in the set \((0, \mu^{-1})\). Thus, \(\Phi\) will range in the set \([0,1)\) with \(\Phi = 0\) only if \(\lambda_i = 0\).

This means that the only constraint imposed upon the distribution criterion is that as long as both factors are present in the country they will both be taxed.\(^5\) The resulting per capita taxation for skilled and unskilled labour in country \(I\) is
\[
t_{sI}(z) = 2(1 - \Phi) (1 - \mu) \quad \text{and} \quad t_{mI}(z) = \Phi (1 - \lambda_i \mu)\]
respectively. Finally, the government budgetary policies must obey the following flow constraint and transversality condition:

\[
\dot{B}(z) - r(z)B(z) + T_i(z) = 0 \quad (4)
\]

\[
\lim_{z \to \infty} B(z) \exp \left[ \int_z^\infty r(\mu) d\mu \right] = 0. \quad (5)
\]

Immigrants are indexed by their vintage \(\xi\) of arrival in their new residence country. Aggregate variables in \(i\) are denoted by upper case letters and can be computed by use of the general aggregation formula (6):

\[
X_i(z) = \frac{1 - \mu}{2} x_i(z) + x_i(0) \lambda_i(0) \mu - \int_0^z g_j(\xi, z) m_{ij}(\xi) d\xi \quad (6)
\]

where \(m_{ij}(z)\) is the flow of migrants from \(i\) to \(j\) at time \(z\), and \(g_j(\xi, z)\) is the value of the variable associated with the individual migrated to \(j\) at time \(\xi\) as of time \(z\). World aggregates are denoted by upper case letters without any subscript and are easy to compute. In particular the aggregate expenditure dynamics and the aggregate resource constraint are:

\[
\dot{E}(z) = \left[ r(z) - \rho \right] [D(z) + H(z)], \quad (7)
\]

\(^5\) The assumption of exogenous distributional equity is adopted also in Baldwin (2000) who assumes \(\varphi = 1/2\); and in Ludema and Wooton (2000) as well as in Kind et al. (2000) who assume that only the mobile factor is taxed (\(\varphi = 1\)).
\[ Y(z) = E(z) = n_1 p_{11} + n_2 p_{22} + A; \quad (8) \]

where \( Y_i = ((1 - \mu) / 2)w_{Ai} + \lambda_i \mu v_{Mi} \) is gross domestic product of country \( i \).

The market equilibrium equations are:

\[
\frac{w^{1-\sigma}_{M1} Y_1}{\lambda_i w^{1-\sigma}_{M1} + \psi^{1-\sigma}_{M1}(1 - \lambda_i)} w^{1-\sigma}_{M2} + \frac{\psi^{1-\sigma}_{M2} Y_2}{\lambda_i w^{1-\sigma}_{M2} + \psi^{1-\sigma}_{M2}(1 - \lambda_i)} w^{1-\sigma}_{M2} = w_{M1}, \quad (9)
\]

\[
\frac{\psi^{1-\sigma}_{M1} w^{1-\sigma}_{M1} Y_1}{\lambda_i w^{1-\sigma}_{M1} + \psi^{1-\sigma}_{M1}(1 - \lambda_i)} w^{1-\sigma}_{M2} + \frac{w^{1-\sigma}_{M2} Y_2}{\lambda_i w^{1-\sigma}_{M2} + \psi^{1-\sigma}_{M2}(1 - \lambda_i)} w^{1-\sigma}_{M2} = w_{M2}, \quad (10)
\]

In this economy there is no physical capital accumulation, therefore expenditure jumps immediately at its steady state value. Further, since the aggregate world income \( Y \) is constant, world aggregate expenditure must be constant, which implies that the interest rate is constantly equal to the subjective discount rate. Equations (9) and (10) determine implicitly the wage rate in each country as function of the distribution of skilled labour between countries; these functions are \( w_{M1} (\lambda_i(z)) \) and \( w_{M2} (\lambda_i(z)) \).

Skilled labour migrates according the present value of the indirect utility differential between countries. If a skilled worker stays in country \( i \) for the rest of her (infinite) life, the discounted value at time \( z \) of her felicity function - denoted by \( v_i(z) \) - is:

\[
V_{Mi}(z) \equiv \int_0^z \left[ \frac{w_{M1}(s) - t_{M1}(s)}{[P_i(s)]^\sigma} \right] e^{-\rho(s-z)} ds,
\]

where \( P_i \) is the price index implied by the felicity function. Skilled workers, however, may migrate. The value of migrating to country \( j \) is the discounted value of the felicity differential between countries, this is: \( v_j(z) \equiv V_{Mj}(z) - V_{Mi}(z) \). The value of migration to \( j \) must equal the cost of migration at any time. It is convenient to assume that
moving costs take the form $\frac{\gamma \lambda_1}{\lambda_1, \lambda_2}$, where $\gamma$ is a positive constant.\(^6\) Equating the value of migration to moving costs at any time gives the differential equation that governs the dynamics of the model:

$$\dot{\lambda}_1(z) = -\frac{\lambda_1(z) \lambda_2(z)}{\gamma} v_{12}(z).$$

(12)

Most of the new economic geography literature has utilised the assumption of static expectations.\(^7\) Under this assumption individuals do not know how wages and prices evolve as function of the state variable. That is, they ignore the implicit functions $w_{M1}(\lambda_1(z))$ and $w_{M2}(\lambda_1(z))$. In this paper we assume rational expectations. In particular we assume that the government inter-temporal budget constraint is fully understood by individuals and it is incorporated into their own. However, to make the analysis comparable with most of the results in the tax competition literature we assume in the main text of the paper that information about the model of the economy is imperfect. Thus, because of insufficient information, individuals utilise current wages as best predictors of future wages. The expected value of $v_{M1}(z)$ is:

$$E[v_1(z)] = \iint_{\mathbb{R}} w_{M1}(z) t_{M} \left( \left[ \frac{w_{M1}(z) - t_{M}(z)}{P(z)^\rho} \right] e^{-\rho(z-x)} dz, (13)$$

where $E$ is the expectation operator. While the assumption of imperfect information concerning the evolution of wages is not unreasonable, in the appendix to this paper we assume perfect foresight and show that the fundamental results remain unchanged. Lastly, we assume that the migration process is anonymous, at any point in time there

\(^6\) Any form of moving costs that gives rise to a differential equation $\dot{\lambda}_1(z) = -f(v_{12}(z))$ where $f$ is a sign preserving function and $f(0) = 0$ could be utilised without the results being affected. The specification in the text, utilised in Baldwin (2001), is analytically convenient and is consistent with the assumption that migration costs increase with the number of people in transit. It can be thought as a queuing system with random selection.
are $\lambda_i(z)$ “tickets” available to country two and the migrants are chosen randomly. Since individuals are all identical the anonymity of the migration process is irrelevant to the results.

2. Inter-temporal tax reallocation and migration.

Trade costs play an important role in the Core-Periphery model. It is however interesting to start the analysis of the debt policy from the case of zero trade costs, so that there are neither agglomeration nor dispersion forces. Further, in free trade $w_{M1}(\lambda_1(z)) = w_{M2}(\lambda_2(z)) = 1$ at any time and therefore current wages are perfect predictors of future wages, thus perfect foresight coincides with rational expectations. The analysis at zero trade costs is interesting because it allows us to illustrate clearly the relation between inter-temporal tax reallocation and migration.

Consider an initial situation where government debt is zero in both countries and Government 2 keeps its budget balanced at any time. Consider then an inter-temporal reallocation of taxes in country 1, keeping government expenditure constant. More precisely, consider a decrease in tax of magnitude $\alpha$ at time $z_0$ associated with an increase of magnitude $\beta$ at time $z_1$. Naturally, $\alpha$ and $\beta$ cannot be set independently for they must be compatible with solvency of government 1 at any time. Using (4) and (5) gives the solvency-compatible relationship between the two constants, which is:

\[ \beta = -\varphi \alpha e^{r(z_1 - z_0)} > 0. \]

Using (11) and the definition of $v_{ij}(z)$ gives:

\[ v_{12}(z) = -\varphi \alpha e^{r(z-z_0)} > 0 \quad \forall z \in [z_0, z_1), \quad (14) \]

\[ v_{12}(z) = 0 \quad \forall z < z_0, \quad \text{and} \quad \forall z \geq z_1. \quad (15) \]

\footnote{See Fujita, Krugman and Venables (1999) for instance. After Krugman, (1991b), the first papers to re-explore the location dynamics under perfect foresight were Baldwin (2001) and Ottaviano (2001).}
Expressions (14) and (15) show that – between $z_0$ and $z_1$ – individuals in country 1 have a window of opportunity to withdraw their shoulders from the burden of debt repayment by migrating to country 2. Government bonds are net wealth for those residents of country 1 who manage to move to country 2 before $z_1$. Government bonds are, instead, a net burden for those left behind. Wealth and consumption of the latter people decreases while that one of the former increases. The wealth increase of the migrant at time $z$ is indeed \( v_{12}(z) \), which corresponds to the discounted value at $z$ of the per capita tax increase the migrant will avoid paying to government 1 at $z_1$. Since migrants will not bear the tax increase at $z_1$, the savings they had accumulated for that purpose (in the form of government bonds) become net wealth. Likewise, part of the bonds held by those who are in country 1 at $z_1$ become a net burden. As tax-payers migrate, the current account of country 1 turns to a surplus, but it goes back to balance at $z_1$ when debt repayment results in a net transfer of resources from residents of 1 to their former fellow citizens. Between $z_0$ and $z_1$ wealth, consumption and GDP of the indebted country decline while those of the other country increase. World aggregate expenditure remains constant and, since world output is also constant, the interest rate remains constant at all times. Migration has an interesting consequence on the transmission of fiscal policy. In the presence of migration the fiscal deficit is transmitted positively to the rest of the world. This is in contrast to all other models exhibiting debt non-neutrality where the fiscal deficit is instead transmitted negatively to the rest of the world and the current account deteriorates. Naturally, debt neutrality would hold if tax-payers were not internationally mobile. We can then establish the first result.
**Proposition 1.** An inter-temporal reallocation of taxes (i.e., a temporary divergence of debt-ratios) induces international migration and results in “quasi-neutrality” of government debt.

Government debt is *quasi-neutral* because the inter-temporal tax reallocation is neutral on the interest rate but it is not neutral on individuals’ wealth, on the current account, and on the spatial distribution of output. It may be interesting at this point to compare this result with those of the macroeconomic literature on debt and output fluctuations. In the models where debt is neutral an inter-temporal tax reallocation is inconsequential on the current account and fiscal shock is not transmitted internationally, i.e., it is neutral on the spatial distribution of output (Frankel and Razin, 1985). In models where debt is not neutral (for instance, because of finite horizon) an inter-temporal reallocation of the tax burden results in an increase of the interest rate and in a worsening of the current account of the indebted country, further the fiscal shock is transmitted negatively to the rest of the world (Blanchard, 1985; Frankel and Razin, 1986). In our model, like in non-neutral macroeconomic models, an inter-temporal tax reallocation is non-neutral on the current account and on the output fluctuations but the effects go in opposite direction (current account surplus and positive international transmission of the shock). This difference is simply due to factor migration.

The consequence of the temporary debt divergence on the spatial distribution of output is readily found. The transitional dynamics can be explicitly found thanks to the fact that for zero trade costs equation (12) is a Bernoulli differential equation whose solution is:
\[
\lambda_i(z) = \left[ 1 + \exp \left( -\varphi \alpha e^{\rho(z-z_0)} \frac{e^{\eta(z-z_0)} - 1}{\gamma \rho} \right) \right]^{-1} \quad \forall z(z_1, z_0)
\]  

(16)

Interestingly, expression (16) shows that \( \lambda_i(z) \) approaches zero in infinite time and therefore any \( z_1 \) is compatible with solvency. This means that temporary budgetary divergence induces permanent migration but this migration will never be such that all skilled labour migrates to country 2. How much skilled labour will have migrated to country 2 at \( z_1 \) depends on the cost of migration (\( \gamma \)), on the size of the initial tax reduction (\( \alpha \)), on the parameter of distributional equity (\( \phi \)) and on the time length between the tax cut and the tax increase (\( z_1-z_0 \)). At \( z_1 \) migration stops because the present value of the tax differential is again zero. The inter-temporal reallocation of taxes creates only a temporary divergence of debt ratios between the two countries, after \( z_1 \) debt is zero in both countries again. This temporary divergence of debt ratios, however, has permanent consequences on the international allocation of the mobile factor.

The tax levels are the same between countries at all times except at \( z_0 \) and \( z_1 \). It is the expectation of future tax liabilities that motivates current migration and not the current tax level differential. *This is interesting because it shows that relaxing the assumption of balanced budget reveals the inter-temporal motive for migration, which remains ignored in balanced budget.* When the debt ratios diverge – other things equal – even if the current tax levels are the same there is migration because diverging debt ratios imply different future tax liabilities which are taken into account in current migration decisions. This leads to the second important conclusion.

**Proposition 2.** Harmonisation of (current) tax rates without convergences of debt ratios (future tax liabilities) does not necessarily reduce migration flows.
Lastly, we ask what would be the effect on migration if the government had announced the inter-temporal tax reallocation before \( z_0 \). Interestingly, even if the policy were announced before \( z_0 \) – say at time \( z_a < z_0 \) – migration would not start until \( z > z_0 \). The reason is simple: in order to gain from migration an individual has to benefit first from the tax reduction occurring at \( z_0 \). Although migration does not start until the policy is implemented, the value of “tickets” to country 2 can be quoted at the time of the announcement. The value at time \( z \in [z_a, z_0] \) of a “ticket” to country 2 to be exerted within \((z_0, z)\) is \( e^{r(z-z_0)}v_{12}(z) > 0 \).

3. Temporary Debt Divergence, Spatial Cycles and Agglomeration.

We now reintroduce trade costs in the model. Introducing trade costs generates agglomeration and dispersion forces. In the presence of agglomeration and dispersion forces migration does not stop when the debt ratios have converged (i.e., after \( z_1 \)). It is possible that after \( z_1 \) skilled labour returns to country 1 but it is also possible that flow migration to country 2 continue until all the skilled labour has left country 1. The latter case is known as the core-periphery configuration. The reason for this to occur is that, beyond a critical level of emigration from 1, agglomeration forces may take over and continue to drive emigration after \( z_1 \). On the other hand, if emigration does not reach that critical level, dispersion forces may dominate after \( z_1 \); in this case skilled labour is attracted by country 1. In the first case, a temporary divergence of debt-ratios between countries generates a spatial cycle, in the second case it results in the core-periphery configuration. To illustrate the mechanism, consider again an initial situation where government debt is zero in both countries. Government 2 keeps its budget balanced at any time. Government 1 uses the tax function to increase or
decrease taxation over time, always compatibly with solvency. Defining the real wages as $\omega_2 \equiv w_{M2} / P_2^\mu$, $\omega_1 \equiv w_{M1} / P_1^\mu$, defining the real wage differential between 2 and 1 as $\omega \equiv \omega_2 - \omega_1$, using (4), (5), (13) and the definition of $v_{ij}(z)$ gives:

$$v_{12}(z) = \frac{1}{r} \omega - \varphi \omega e^{(z-z_0)} > 0 \quad \forall z \in [z_0, z_1), \quad (17)$$

$$v_{12}(z) = \frac{1}{r} \omega, \quad \forall z < z_0, \text{ and } \forall z \geq z_1. \quad (18)$$

The first term in (17) is the real wage differential, the second term is the future tax liability resulting from debt repayment due at $z_1$. Individuals in country 1 can avoid the burden of debt repayment by migrating to country 2 before $z_1$. The migration of skilled labour induces changes in the real wage rate differential through equations (9) and (10) which then feed back on the value of migration and so on in a circular causation. Between $z_0$ and $z_1$ the balance between the debt burden and the real wage differential drives migration, as shown by equation (17). After $z_1$, only the real wage differential drives migration, as shown by equation (18). The long-run spatial configuration is determined by the size of trade costs, the size of migration costs and the size of budgetary divergence. Figure 1 shows the phase diagram of the dynamic system for intermediate trade costs.

![FIGURE 1](attachment:figure1.png)
The solid line represents the phase line before $z_0$ and from $z_1$ onward. As the consequence of the budgetary shock, the phase line slowly moves upward between $z_0$ and $z_1$ (dashed line), and drops again to its initial position at $z_1$. There are three internal equilibria, each one at the intersection of the phase line with the horizontal axes. Henceforth, we will refer to the equilibrium in the middle as the Central spatial configuration ($\lambda_C$) and we will refer to the two lateral equilibria as the Left and the Right spatial configuration ($\lambda_L$ and $\lambda_R$). Inspection of the phase line shows that the Central configuration is stable while the lateral configurations are both unstable. There are two more stable spatial configuration that correspond to $\lambda_1=0$ and $\lambda_1=1$. These are called core-periphery configurations. After $z_1$, the set ($\lambda_L$, $\lambda_R$) is the basin of attraction of $\lambda_C$ and the set ($0$, $\lambda_L$) is the basin of attraction of the core-periphery configuration at $\lambda_1=0$. Between $z_0$ and $z_1$, the phase line shifts upward and the two equilibria $\lambda_L$, and $\lambda_C$ get closer to each other and may eventually disappear if the dashed line shifts up enough.

Whether temporary debt divergence results in a spatial cycle or in a core-periphery configuration depends on the value of the state variable at $z_1$. If $\lambda_1(z_1) \in (0, \lambda_L)$, then the system will asymptotically approach the core-periphery outcome. Conversely, if $\lambda_1(z_1) \in (\lambda_L, \lambda_C)$, then the system will asymptotically return to the symmetric equilibrium. The value of $\lambda_1(z_1)$ is smaller the smaller are migration costs and the larger is the size of the tax cut. Using computer simulation it is possible to find the time path of $\lambda_1(z)$ for different values of $\alpha$ and $\gamma$. Figure 2 illustrates two

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8 The system (9) and (10) cannot be solved explicitly for $\omega(\lambda)$. Following Baldwin (2001) we interpolate this function with a continuous polynomial. We use Chebyshev's interpolation method instead of Lagrange as the former performs remarkably better (see Judd, 1998). The approximation has been computed for the following parameter values: $\mu = 0.4$, $\sigma = 3$, $\tau = 0.15$.

9 Given the definition of $\omega$, a positively sloped intersection of the phase line with the horizontal axis means a stable equilibrium.
representative cases. On the vertical axes we have $\lambda$ and on the horizontal axes we have time. In both simulations $z_1 - z_0 = 32$ and the inter-temporal rate of time preferences was set at 5%. The dashed line represents a case of high migration costs while the dot-dashed line represents the case of low migration costs. In the simulations the values assigned to $\gamma$ were 1.5 and 1.2 respectively thus simulating a 20% drop in migration costs. The value assigned to $\alpha$ was −10% of the initial GDP. The distributional parameter $\varphi$ was set at $\varphi = 0.8$. The solid horizontal line intercepts the vertical axes in correspondence of $\lambda_L$ (the same value as in Figure 1). For low migration costs $\lambda(z_1) \in (0, \lambda_L)$ (dot-dashed line) therefore - after the equality of debt ratios is restored - the system will continue on an unstable path asymptotically approaching the core-periphery configuration. For high migration costs $\lambda(z_1) \in (\lambda_L, \lambda_R)$ and, after the equality of debt ratios is restored, the system asymptotically returns to its initial equilibrium (the Central configuration). Similar patterns would emerge if, keeping migration costs constant, we considered large and small inter-temporal tax reallocations respectively.

![Figure 2](image.jpg)

**FIGURE 2**

The result of this analysis is that temporary divergence of debt ratios generates a spatial cycle or a core-periphery outcome. The amplitude of the cycle and the likelihood of the core periphery configuration decreases with $\gamma$ and increase with $|\alpha|$.
and $z_1 - z_0$. The values of $|\alpha|$ and $z_1 - z_0$ determine the amplitude of debt divergence.

Let us define with $b_{h, \text{div}}$ the maximum divergence of debt ratios such that leaves $\lambda(z_1)$ in the basin of attraction of $\lambda_C$ and let us call it the “break point debt divergence”. A temporary divergence of debt ratios beyond the “break point debt divergence” results in core-periphery configuration. We can then summarise the results of this section as follows:

**Proposition 3.** A temporary divergence of debt ratios causes a spatial cycle or a core-periphery configuration. The “break point debt divergence” is smaller the smaller migration costs.

This result suggests that a large, albeit temporary, divergence of debt ratios may have permanent consequences on the spatial distribution of the mobile factor. Small temporary divergences of debt ratios are not free from insidious complications however. Indeed, if trade costs or migration costs fall during the adjustment process, what initially could have been a spatial cycle might result in a core-periphery configuration. Since changes to trade costs and to migration costs may be unexpected and out of the control of governments, even small and temporary budgetary divergences may have undesired consequences.

If divergence of debt ratios is not recommendable, should converge be? Interestingly, the answer is not necessarily positive.

4. Debt Convergence, Spatial Cycles and Agglomeration.

Consider an initial situation of stable but different debt ratios in the two countries. Without loss of generality it is assumed that the debt ratio is equal to zero in
country 2 and positive in country 1. Consistency with solvency requires that the tax rate be higher in country 1 because of the need to service the debt, that is: $T = rB$. The difference in the tax rate has induced migration in the past, which shifted the central spatial configuration to the left of $\frac{1}{2}$. It is assumed that the economies have already reached this stable spatial configuration when the policy of debt convergence is announced. Formally, this means to assume that $B(z)/Y(z) = \overline{B} > 0$ (where $\overline{B}$ is a constant) and that $\lambda(z) = 0$, for $z \in (z_0 - \delta, z_0)$, with ($\delta > 0$), where $z_0 - \delta$ is the last time that the system has reached a steady state and $z_0$ is the time when the policy of convergence is announced.\(^{10}\) The thin line in Figure 3 represents the phase line at $z_0$.

For this example the debt ratio was set at 90% and the distributional parameter is $\varphi = 8$. Consequently, $\lambda(z_0) \approx 0.363$, which is to the left of $\frac{1}{2}$.

![Figure 3](image_url)

**FIGURE 3**

At $z_0$ governments commit to a “stability” pact that requires the debt ratio be non larger than $\overline{b} < \overline{B}$ by $z_1$. In the numerical computation we set $\overline{b} = 60\%$. The initial equilibrium is now disturbed by the additional tax burden that the commitment to convergences imposes on tax-payers in country 1. In order to achieve convergence

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\(^{10}\) To be precise, since the adjustment is asymptotic, $z_0 - \delta$ is the last time the system was arbitrarily close to the steady state.
government $I$ must increase taxation sometime between $z_0$ and $z_1$. In order to avoid the expected future tax liability, skilled labour migrates to country $2$. The value of migration reflects the burden debt repayment. Using (4), (5), (13) and the definition of $v_y(z)$ we have:

$$ v_{12}(z) = \frac{1}{r} \omega + \phi B_1(z) = 0, \quad \forall z \in (z_0 - \delta, z_0), \quad (19) $$

$$ v_{12}(z) = \frac{1}{r} \omega + \phi B_1(z) + \phi [B(z_1) - \bar{B}(z_1)] e^{-r(z_1 - z)} > 0 \quad \forall z \in [z_0, z_1), \quad (20) $$

$$ v_{12}(z) = \frac{1}{r} \omega + \phi B_1(z) \neq 0 \quad \forall z \geq z_1. \quad (21) $$

These expressions can be inspected in parallel to Figure 3. The thick line represents the phase line after convergence is achieved. The thin line represents the phase line that obtains by use of (19). Between $z_0$ and $z_1$ the phase line moves upward and it can be obtained by use of (20). At $z_1$ the phase line drops down and is represented by the thick line, which results from (21). The long run spatial configuration depends on the value of $\lambda_1(z_1)$. If $\lambda_1(z_1)$ is to the left of $\lambda_L$ on the thick line, then the system will asymptotically approach the core-periphery configuration. Conversely, if $\lambda_L < \lambda_1(z_1) < \lambda_R$ on the thick line, the system will asymptotically approach the central spatial configuration $\lambda_C$ on the thick line. The value of $\lambda_1(z_1)$ is smaller the smaller are migration costs and the larger is the difference $\bar{b} - \bar{b}$. Using numerical methods it is possible to find the path of $\lambda_1(z)$ for different values of $\bar{b}$ and $\gamma$. Figure 4 illustrates the cases of high and low migration costs; again in the simulations the values assigned to $\gamma$ were 1 and 0.9, thus simulating a 10% drop in migration costs. In both cases $z_1 - z_0 = 6$. The dashed line represents the time path of the state variable in the case of high migration costs (a spatial cycle) while the dash-dotted line represents the case of small migration costs (the core-periphery case). In
the stable case the economy reaches the new central spatial configuration on the thick line of Figure 3 closer to $\frac{1}{2}$. A similar pattern obtains by computing high and low $\bar{b}$ respectively.

![FIGURE 4](image)

Figure 4 represents a general pattern found with simulations: the amplitude of the cycle increases with the tightness of budgetary converges (low $\bar{b}$) and with the inverse of migration costs (low $\gamma$). This leads us to formulate the following proposition.

Let us define with $b_{b,\text{conv}}$ the maximum reduction of the debt ratio that leaves $\lambda(z_1)$ in the basin of attraction of $\lambda_c$ (on the thick line) and let us call it the “break point debt reduction”. That is $b_{b,\text{conv}} = \max(\bar{b} - \bar{b})$ such that $\lambda(z_1) \in (\lambda_c, \lambda_r)$. We can summarise the results as follows.

**Proposition 4.** Debt convergence generates a spatial cycle or a core-periphery outcome. The “break point debt reduction” is smaller the smaller migration costs.

These results suggest that convergence of debt ratios should be achieved with caution because it may induce migration of the mobile factor and, with it, migration of
the tax base. Too ambitious budgetary targets (low \( \bar{b} \)) induce larger cycles and increase the likelihood of the core-periphery configuration. Further, unexpected declines in the transport or migration costs may deteriorate the situation by transforming a cycle into a core-periphery configuration.

The results also suggest that economic integration renders the achievement of debt convergence increasingly difficult. This is because the fall of trade or migration costs causes an increase in the amplitude of the cycle and in the likelihood of the core-periphery configuration. Then, it would seem safer that debt convergence precede economic integration rather than follow it.

5. Debt convergence, divergence, and the stability of the Core-Periphery configuration.

This section illustrates the consequences of an inter-temporal tax reallocation when the indebted country has the core.

We start by computing the “sustain point debt gap”. Consider a positive and constant level of debt in country 1 and no debt in country 2. The “sustain point debt gap” is the maximum value of the constant debt-ratio differential that does not destabilise the CP. Assume that country 1 has the core; the real wage differential is \( \omega(1) < 0 \) which can easily be computed from equations (9)-(10). Utilising the expression for GDP in each country we obtain the sustain point debt gap. This is:

\[
b_{s,sp} = \frac{-2\omega(1)}{(1 + \mu(2\rho))}.
\]

If the constant level of the debt ratio differential is larger than \( b_{s,sp} \) the core-periphery configuration is unstable and country 1 looses the core after a small shock.
Let us now consider an inter-temporal reallocation of taxes. In this case, the debt ratio can temporarily be larger than the sustain point debt gap without country 1 necessarily loosing the Core. We are interested in finding the “sustain-point temporary debt gap”, that is the maximum inter-temporal tax reallocation that does not make country 1 lose the core. If the Core country is not to lose the core, the maximum inter-temporal tax reallocation must be such that $\lambda$ does not leave the basin of attraction of the core-periphery configuration. The value of $\lambda$ at any time depends on the initial tax shock ($\alpha$) on the time length of the inter-temporal tax reallocation ($z_1 - z_0$) and on the cost of migration ($\gamma$). Consider an inter-temporal tax reallocation operated by country 1 (the core country). Taxation is reduced at $z_0$ and increased at $z_1$ consistently with the inter-temporal budget constraint. The debt level increases between $z_0$ and $z_1$, and there is a time $z_*$ at which the debt has reached the level $b_{s, cp}$. Beyond this level of debt ratio migration starts towards country 2 but whether country 1 loose the core in the long run depends on whether $\lambda$ at $z_1$ is inside or outside the basin of attraction of the core-periphery configuration. The value of $\lambda(z_1)$ depends on the speed of migration, on the magnitude of the inter-temporal tax reallocation and on its length. Figure 5 shows the simulations for the case of a brief and a long temporary debt divergence.

![Figure 5](image-url)
In the first case (dashed line) \( \lambda(z_1) \) is in the basin of the attraction of the CP and therefore country 1 does not lose the core in the long run. In the second case (dash-dotted line) \( \lambda(z_1) \) is outside the basin of attraction of CP and country 1 loses the core. There is a critical time \( z_{s,bacp} \) such that \( \lambda(z_{s,bacp}) = \lambda_R \). Any inter-temporal tax-reallocation longer than \( z_{s,bacp} - z_0 \) brings \( \lambda \) outside the basin of attraction of the core-periphery (bacp). The critical length of the inter-temporal tax reallocation is the sum of two sub-periods: \( z_{s,bacp} - z_0 = (z_{s,cp} - z_0) + (z_{s,bacp} - z_{s,cp}) \). The first sub-period is the time it takes for the debt ratio to reach the level \( b_{s,cp} \). This sub-period can be computed and it is: \( z_{s,cp} - z_0 = \ln(\frac{\alpha(1)}{-\alpha\phi}) \). The second sub-period depends on \( \alpha(\lambda(z)) \) and \( \gamma \). It can only be approximated numerically (in the examples of Figure 5 \( z_{s,bacp} - z_{s,cp} = 10.8 \)).

A pair of results similar to those shown in Figure 5 would obtain if we considered respectively high and low migration costs for any given \( \alpha \) and \( z_1 - z_0 \). Also, a similar pair of results would obtain if we considered a small and large \( \alpha \) for any given \( \gamma \) and \( z_1 - z_0 \).

The last question we ask concerns debt convergence. Assume that country 1 has a stable debt ratio equal to \( b_{s,cp} \). If country 1 commits to a reduction of the debt ratio to \( \overline{b} < b_{s,cp} \) by \( z_1 \), expected taxation increases between \( z_0 \) and \( z_1 \). After \( z_1 \) the CP configuration is indeed stable because \( \overline{b} < b_{s,cp} \) but \( \lambda \) may already have left the basin of attraction of the core-periphery configuration. Similarly to the case of temporary debt divergence, in the case of debt convergence what matters for the long run
configuration is the value of $\lambda$ at $z_1$. There is a critical value $z_{su,conv}$ such that $\lambda(z_{su,conv}) = \lambda_R$. It is intuitive that $z_{su,conv} - z_a = z_{s,bacp} - z_{s,cp}$ (where $z_a$ is the time when the convergence policy is announced).

These two results can be summarised in the following proposition.

**Proposition 5.** Temporary debt divergence may result in the break up of the CP configuration. The “sustain-point temporary debt-gap” is small if factors markets are more integrated, if the reallocation interval is large, and if the initial tax reduction is large. The same, mutatis mutandis, applies to debt convergence.

6. Implications for tax competition.

It is interesting to draw some implications concerning the analysis of tax competition in balanced budget. When balanced budget is assumed, any tax change must be accompanied by a compensating change in government expenditure such that the level of debt remains at its constant level (usually assumed to be zero). Abstracting from the realism of this assumption we ask what are the consequences of removing it on the ability of a country to engage in tax competition. In particular, we consider a Core country and we focus on the consequences of removing the assumption of balanced budget on its ability to defend the core.

Let us define with $\Delta t_s$ the maximum tax gap that keeps the CP stable. Using the concept introduced in Baldwin et al. (2003) we can call this gap the “sustain point tax gap”. Barrowing again from these authors we can call the tax level that extracts the maximum agglomeration rent the “limit tax”. The set $(0, \Delta t_s)$ gives the set of all permanent tax gap that are compatible with stability of the CP configuration.
Therefore, for any given tax level in the periphery, $\Delta t_s$ measures the size the CP-stable strategy set at disposal of the core country. The value of $\Delta t_s$ measures the ability of the core country to engage in tax competition and its ability to defend the core. We want to study the consequences of removing the balanced budget assumption on the size of this strategy set.

Consider first the case in which debt is zero in both countries. In such case $\Delta t_s = \omega_1(1) - \omega_2(1)$. Consider then a case of permanent but positive debt ratios in each country. It is immediate that the value of $\Delta t_s$ is reduced if the core country has a larger debt ratio than the peripheral country. Assume that country 1 and 2 have a stable debt ratio equal to $b_1$ and $b_2$ respectively and that $b_1 > b_2$. Then, we have that $\Delta t_s = \omega_1(1) - \omega_2(1) - (\tau_1 - \tau_2)$ where the term in parenthesis represents the current and permanent real tax level differential between the two countries that result from the need to service the debt. It is clear that the set of CP-stable strategies has shrunk and that the limit tax has become smaller. A larger debt ratio reduces the core country’s ability to defend the core.

Let us now consider an inter-temporal tax reallocation, i.e., a temporary debt divergence. Consider an inter-temporal tax reallocation such the debt ratio remains smaller than $b_{s, cp}$. The sustain tax gap is:

$$\Delta t_s(z) = \omega_1(1) - \omega_2(1) - (\tau_1 - \tau_2) - \theta(z) \quad \text{for } \forall \ z \in (z_0, z_1),$$

(22)

where $\theta(z)$ is the expected present value of future taxation. The set of strategies at disposal of the core country is temporarily reduced by an inter-temporal tax reallocation. Therefore, an inter-temporal tax reallocation reduces temporarily the

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11 If we continue to assume for simplicity that country two has no debt, then $\tau_1 = \varphi \rho B / P^\mu_1$ and $\tau_2 = 0$. **
ability of the core country to defend the core and it makes it temporarily a weaker player in tax competition.

If we consider a policy of debt reduction we have that:

\[ \Delta \tau_1(z) = \omega_1(l) - \omega_2(l) - (\tau_1 - \tau_2) - \theta(z) \quad \forall \ z \in (z_0, z_1) \]  

(23)

and

\[ \Delta \tau_1(z) = \omega_1(l) - \omega_2(l) - (\bar{\tau}_1 - \tau_2) \quad \forall \ z > z_1. \]  

(24)

Since \((\bar{\tau}_1 < \tau_1)\) it follow that the policy of debt reduction will expand country 1’s strategy set after \(z_1\), but during the transition period the strategy set becomes smaller.

These results can be summarised as follows:

**Proposition 6.** An inter-temporal tax reallocation or a policy of debt reduction weakens temporarily the core’s country ability to defend the core and to engage in tax-competition.

The balanced-budget tax competition analysis neglects the terms \((\tau_1 - \tau_2)\) and \(\theta(z)\). Neglecting the constant term \((\tau_1 - \tau_2)\) obscures the fact that, other things equal, large-debt core countries are weaker player in tax competition and that the limit tax is smaller than what it would be in balanced budget. Neglecting the term \(\theta(z)\) obscures the fact that, other things equal, a large core-country’s ability to engage in tax competition may be reduced by the need to reduce the debt. The role of expected taxation is particularly important when countries have committed to policies of debt reduction or when they operate asymmetric inter-temporal tax reallocations (such as countercyclical fiscal policies following asymmetric shocks).
Proposition 6 suggests a number of interesting implications. Large countries with high debt or large countries undergoing a debt-reducing policy should, other things equal, be averse to tax competition because their strategy set is (temporarily) smaller and their ability to defend the core is reduced. In contrast, small countries with low debt should be against tax harmonisation because they have a bigger chance of “snatching” the core. A country’s aversion to tax competition may change over time. Once an inter-temporal tax reallocation is completed or once debt convergence is achieved, a country may become more favourable to tax competition. A further implication concerns the ability of a country to stick to commitments on debt policies. Because of fierce tax competition, a large-core country may be constrained on its ability to reduce debt by its desire to keep the core.

Conclusion

The paper has studied the effects of the debt divergence and convergence on the spatial distribution of output. The analysis departs from the tax competition literature by relaxing the assumption of balanced budget. In this inter-temporal context migration is motivated by the desire of avoiding future tax liabilities.

An immediate consequence of relaxing the balanced budget assumption is that tax harmonisation does not necessarily reduce migration of factors unless it is done in an environment where debt ratios are not diverging (proposition 2). Clearly, convergence of current tax rates does not reduce migration if future tax liabilities diverge. Another interesting result, which relates to the Ricardian analysis of public debt, is that in the presence of migration an inter-temporal tax reallocation is “quasi neutral” and causes a spatial reallocation of output away from the indebted country (proposition 1).
When agglomeration and dispersion forces are present in the model debt policies have some other consequences. An inter-temporal reallocation may generate a spatial cycle or a core-periphery configuration. There is a limit to the divergence of debt ratios called the “break point debt divergence” such that any divergence larger that this one results in the core periphery configuration. An interesting aspect of the break point debt divergence is that it is smaller the lower migration costs (Proposition 3). The second policy analysed is that of debt convergence. Somewhat surprisingly a policy of debt convergence may also be destabilising. Convergence of debt ratios implies an increase in future tax liabilities for individuals in the debt-reducing country and this induces migration of the mobile factor. There is a limit to the magnitude of debt convergence, called the “break point debt reduction”, beyond which the core-periphery configuration occurs. Interestingly, the “break point debt reduction” is smaller the lower are migration costs (proposition 4).

The Growth and Stability Pact puts limits to debt divergence and sets targets in terms of convergence of debt ratios. The two results just discussed shows that policies that restrain debt divergence have indeed a stabilising effect on the spatial distribution of output. Conversely, policies of debt convergence, if too ambitious, may have destabilising effects. A further implication derives from the numerical example. The example, which is representative of many numerical simulations, has shown that an inter-temporal reallocation of taxes that creates an initial deficit differential of 10% of GDP destabilised the economy only after 32 periods. Conversely, a debt convergence from 90% to 60% of GDP destabilised the economy already after 6 periods. If we had to draw implications for the Stability Pact it would be that counter-cyclical deficits are not a threat for stability whereas too ambitious programmes of debt convergence may be. Perhaps surprisingly, a policy of debt convergence may be more destabilising than
a policy of debt divergence. The result mentioned above also suggests an ideal order of moves into economic integration. Since debt convergence is more likely to be destabilising when migration costs are low, it would seem safer that debt convergence be achieved before the integration of labour markets rather than after.

The analysis at the core-periphery has shown that this spatial configuration becomes unsustainable when the debt gap is larger than the “sustain point temporary debt gap” or when the convergence policy is too ambitious (proposition 5). The paper also asks what implications for the balanced-budget tax-competition literature emerge from this analysis. It has been shown that debt divergence and debt convergence, because of the future tax liability they imply, reduce the set of core-sustainability-compatible permanent tax level available to the core country. That means that the “limit tax” becomes smaller, which makes the core country a weaker player in a tax competition game. This would seem to imply that large countries with high debt should be averse to tax competition while small countries with small debt should be averse to tax harmonisation.
Appendix: Perfect foresight.

We have already seen that in the absence of trade costs real wages are constant. Therefore, whether individual have perfect information about future wages or, instead, only know and use current wages to predict future wages is irrelevant. Thus, assuming perfect foresight (rational expectation plus perfect information) does not affect Propositions 1 and 2. Only Propositions 3-6 are potentially affected by the assumption of perfect foresight. With perfect foresight the dynamic system becomes:

\[
\dot{v} = r\dot{\nu} - \omega - r\varphi(\omega(z) - z) (1 - u_z(z)) \quad (A.1)
\]

\[
\dot{\lambda} = -\frac{\lambda(1 - \lambda)}{\gamma} \nu \quad (A.2)
\]

where \(u_z(z)\) is a unit step function and \(\dot{\nu}\) is the discounted value of \(\omega\). It may be useful to note that, if we neglect the last term in (A.1), the system is isomorphic to the system in Ottaviano (2001) and Baldwin (2001). The main feature of this system is that for certain constellations of parameters it exhibits multiple saddle paths.

The effect of debt divergence on the dynamic system is qualitatively the same as illustrated in the text: future tax liabilities, represented by the last term in (A.2), acts like a forcing function to the system shifting it smoothly between \(z_0\) and \(z_1\); at \(z_1\) the system returns to its position before \(z_0\). The effect of the forcing function is simply to shift temporarily all the phase trajectories. The long run equilibrium then depends on which saddle path will be reached at \(z_1\). This depends crucially on the position reached by \(\lambda\) at \(z_1\) and on the size of migration costs. Three cases may emerge.  

12 See Baldwin (2001) for the analysis of these three representative cases (without future tax liabilities).
Large migration cost. In this case there is no overlap of the saddle paths. The jumping variable $\nu$ will jump on a saddle path leading to $\lambda_C$ if $\lambda_i(z_i) \in (\lambda_L, \lambda_R)$. Instead, if $\lambda_i(z_i) < \lambda_L$ or $\lambda_i(z_i) > \lambda_R$, then $\nu$ will jump on a saddle path that leads to a core-periphery outcome. There is no ambiguity about which long-run spatial configuration will emerge, it all depends on the value of $\lambda_i(z_i)$. Only history matters for large migration costs. In this case, the dynamics of perfect foresight is qualitatively equivalent to the dynamics of rational expectations with imperfect information. Thus, all Propositions 3-6 remain valid.

Intermediate migration cost. In this case there is overlap of saddle paths in a neighbourhood of $\lambda_L$ and in a neighbourhood of $\lambda_R$. The neighbourhoods in which there is overlap of saddle path are known as the “regions of overlap” (Krugman, 1991b). If $\lambda_i(z_i)$ is in anyone of these regions, then the long run equilibrium will be $\lambda_C$ or the core-periphery outcome depending on which saddle path is chosen. Instead, if $\lambda_i(z_i)$ is outside these regions, then the long run equilibrium will be $\lambda_C$ if $\lambda_i(z_i) \in (\lambda_L, \lambda_R)$ while it will be the core-periphery outcome if $\lambda_i(z_i) < \lambda_L$ or $\lambda_i(z_i) > \lambda_R$. The overlap of saddle paths generates indeterminacy because the same “initial condition” $\lambda_i(z_i)$ may lead to either one of two possible long-run spatial configurations. Both history and expectations matters in the case of intermediate migration costs. Outside the overlap regions the results of the paper apply the same. In the overlap regions there is indeterminacy. The statements in Propositions 3, 4, 5, and 6 apply outside the “overlap region”. All the critical values of debt (“break points debt divergence”, “break point debt reduction” and “sustain point temporary debt gap”) become smaller.
Small migration cost. The same indeterminacy as in case (2) emerges with the difference that the regions of overlap cover the entire domain (0,1). Therefore, regardless of $\lambda_t(z_t)$, any one of the long-run equilibria may be reached. In terms of the implications for the debt policy this is a limit case for it implies that all the critical values of debt are zero.
REFERENCES


