Abstract
This paper analyses the effect of stricter sanctions against fraudulent disclosure in an economy where commercial lenders have only an imperfect information about the type of the firm they trade with. The model is cast as a game between the firm’s manager and the supplier of an essential commodity unit. On the one hand, when the sanction gets heavier, more managers who run fragile firms will honestly announce the type of the firm and thus face a larger probability of default, since suppliers will charge the higher price. On the other hand, the default premium incorporated into the input price will decline for all managers who claim that their firm is solid, be them at the head of fragile or solid firms. While the two effects tend to offset each other, in this model, the policy brings about an increase in the economy-wide default premium and in the frequency of defaulting firms.

Keywords: Financial distress, Disclosure, Corporate regulation, Bayesian Equilibrium.
JEL Classification: D82, G33, G38, K22

Résumé
Cet article analyse les conséquences d’un alourdissement des sanctions en cas de publication d’informations frauduleuses. Le modèle est développé sous la forme d’un jeu à information imparfaite opposant une firme et son sous-traitant en supposant que les sous-traitants ont une information imparfaite concernant les caractéristiques de la firme pour laquelle ils opèrent. On vérifie que lorsque la sanction se durcit, un nombre plus important de firmes fragiles va adopter une politique de communication honnête. Cependant, parce que leurs fournisseurs exigeront des prix plus élevés, ces firmes devront supporter un risque accru de défaut. Dans le même temps, la prime de défaut exigée par les sous-traitants aux firmes annonçant une forte solidité financière va diminuer. Le risque de défaut diminue donc pour ces firmes – quel que soit leur situation financière effective. Dans notre modèle, la compensation des ces deux effets contradictoires fait apparaître une augmentation globale de la prime de risque de défaut et de la fréquence des situations de faillite.

Mots clés : Risque de faillite,annonce de résultat, régulation industrielle, équilibre Bayésien.
Classification JEL: D82, G33, G38, K22
Strategic managerial dishonesty and financial distress

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Abstract
This paper analyses the effect of stricter sanctions against fraudulent disclosure in an economy where commercial lenders have only an imperfect information about the type of the firm they trade with. The model is cast as a game between the firm’s manager and the supplier of an essential commodity unit. On the one hand, when the sanction gets heavier, more managers who run fragile firms will honestly announce the type of the firm and thus face a larger probability of default, since suppliers will charge the higher price. On the other hand, the default premium incorporated into the input price will decline for all managers who claim that their firm is solid, be them at the head of fragile or solid firms. While the two effects tend to offset each other, in this model, the policy brings about an increase in the economy-wide default premium and in the frequency of defaulting firms.

Keywords: Financial distress, Disclosure, Corporate regulation, Hybrid Bayesian Equilibrium.
JEL Classification: D82, G33, G38, K22

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1 Introduction

After the burst of the US Internet bubble in March 2001, juridical investigations unveiled that in the late nineties several top-level managers manipulated financial information so as to inflate the share value of their company, then cashed the overvalued stocks just before the company collapse. This proliferation of dishonest behavior brought about significant distrust about large public companies and their management. To address this problem, the American administration took a set of regulatory steps, mainly through the Sarbanes-Oxley Act of 2002, which is often referred to as the most important securities legislation since the original federal securities law of the 1930s. One main goal of the new regulation was to enhance the responsibility of chief executives with respect to the truthfulness and relevance of compulsory financial statements.1 Furthermore, changes in the federal sentencing guidelines in 2001 and 2003 significantly raised penalties for financial fraud; economically damaging frauds are now on the same level as armed robberies.

The implicit assumption behind this regulatory change is that “honesty is the best policy” or, in the economists’ jargon, that more information is always better than less. However, we all know from every-day life experience that, depending on the situation, telling the naked truth might not be the most efficient strategy. Sometimes, lies help avoiding an useless conflict. To take a every-day life example provided by Fletcher (1966), a British moral philosopher, it may be wise to keep for yourself telling to your wife that you don’t like her new green dress, even if this is your true opinion. This common sense principle does apply to the world of businesses too. As emphasized by Lev (2003, p.36), “the more common reason for earnings manipulation is that managers, forever the optimists, are trying to ‘weather out the storm’ – that is, to continue operations with adequate funding and customer / supplier support until better times come.” The literature on corporate financial distress has emphasized that the image clients and suppliers have about a company plays an important role in determining its actual financial stance. More precisely, if creditors start having doubts about the financial position of a company, they may ask for a higher

1 Other important goals were to tighten up supervision of the practices of the accounting profession, strengthen auditor independence rules, enhance the timeliness and quality of financial reports of publicly listed companies, and protect employees’ retirement plans from insider trading.
risk premium, which represents an indirect cost for the firm (e.g., Altman, 1984; Wruck, 1990; Andrade and Kaplan, 1998). So, in difficult times the manager may well communicate on better than actual performances only to get more favorable contracting terms and push down these indirect costs. To the opposite, if the manager cannot use the communication weapon freely, his company will be submitted to additional strain. Reduced flexibility in choosing the most appropriate communication strategy might therefore entail an indirect cost of doing business in a decentralized economy.

This paper analyzes the impact of a tougher sanction for fraudulent disclosure on the firms’ survival rate and on the economy-wide default premium. The model is cast as a simple game between the manager – who chooses his communication policy – and the supplier of an essential input, under supplier’s asymmetric information about the type of firm. Both agents are risk-neutral. The supplier acts as a lender, and prices the firm as a residual claimant who breaks even. There are two types of firms, the high and the low expected return firm. The manager must announce the type of the firm before the time when the supplier posts the input price. He may either declare honestly the true type of the firm or lie. The end-of-period revenue of the firm is random. Depending on whether the firm may reimburse the trade credit, it survives or it is pulled out of the market. In general, firms that become bankrupt come under close public scrutiny. We thus assume that when a firm files for bankruptcy, managers who have disclosed false information are fined (as required by the new US regulation). Our analysis focuses on the Hybrid Bayesian Equilibrium, where at least some of the managers at the head of fragile, low expected return firms choose to disclose false information. When the penalty for fraudulent disclosure goes up, several effects come into operation. As expected, more managers at the head of low return firms will truthfully state that their firm is fragile. They thus have to pay a higher input price and are subject to a larger probability of default. On the other hand, the default premium for managers who claim that their firm is solid goes down, the input price diminishes, and the probability of default of these firms declines. Since the two effects tend to offset each other, determining

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Empirical studies about the cost and benefits of the law are yet scarce. So far, the most important piece of evidence was brought by Zhang (2005, p.2), who argues that the new regulation might have brought about value losses for US public companies as high as 1400 billion dollars.
the optimal sanction is a difficult question. A clear answer may be provided in the case where
the firms’ income follows a uniform income distribution. In this case, it can be shown that an
increase in the penalty entails a net positive variation in both the overall default premium and
the frequency of defaulting firms. While we cannot falsify this relatively strong conclusion by
means of numerical simulations performed in the case of lognormal distributions, it cannot be rule
out that, for other distributions or other parameter values, the conclusion would be less extreme.
Therefore, the paper message should be inferred not as a plea for removing all sanctions, but as
a call for a careful weighting of the two effects when choosing the sanction weight.

Our conclusion is very much in line with that of Morris and Shin (2004). They show that
increasing the quality of public information might exacerbate the coordination problem among
creditors, and thus foster the bankruptcy rate in a decentralized economy. More in detail, if a
creditor believes other creditors will call their loans, it become individually rational to call his loan.
Hence creditors’ actions are strategic complements. Creditors try to infer what other creditors
believe, as well as the intrinsic quality of the borrower from a publicly available information.
This ‘higher-order beliefs problem’ results in each creditor overweighting public signals relative to
their own private information when making inferences. Consequently, more transparent public
information can result in more frequent default, for a given intrinsic quality of the borrower. This
model shares with Morris and Shin (2004) the idea according to which lenders’ perceptions of
default risk have a real economic impact in that they influence the reality of default. However,
in this paper, the main result is put forward in a more straightforward way, since there is no
coordination problem. The mechanism at work is simple: if the creditor perceives the borrower as
being of the low intrinsic quality with a high probability of default, he will offer less favorable terms
of credit, which in turn increase the probability of default. This paper can also be connected to
studies that analyze the two-person game between an auditee who can commit fraud and an auditor
who can cover this fraud; they emphasize the necessary conditions for fraudulent overstating to
be an equilibrium.³

³ See e.g. Antle (1982), Anderson and Young (1988), Newman and Noel (1989), Yoon (1990), Patterson and
The paper is organized as follows. The next section introduces the basic assumptions. Section 3 presents the equilibrium of the game. The relationships between the level of the sanction the economy-wide default premium and the frequency of defaulting firms is analyzed in Section 4, for the uniform distribution case. Section 5 presents a numerical simulation for a lognormal distribution of the firms’ income. The last section concludes the paper.

2 The model

There is a continuum of firms that each lives one-period. The income of the firm \( i \) is a random variable \( \tilde{y}_i \), following a cumulative distribution \( F^i(\cdot) \) on the support \([0, \tau^i]\).

We assume that there are only two types of firms, the \((H)igh\) and the \((L)ow\) expected return firm. Denoting the expected income by \( \bar{y}^i = \int_0^{\tau^i} y dF^i(y) \), with \( i \in \{H, L\} \), the two types of firms are thus characterized by \( \bar{y}^H > \bar{y}^L \).

The total number of firms is normalized to one; the frequency of \(H\)-type firms in total population of firms is denoted by \( q \), with \( q \in [0, 1] \); this frequency is common knowledge.

In order to produce the final good, the manager of the public company must buy one unit of an essential commodity from an external supplier. We assume perfect competition between suppliers. Information is asymmetric: the manager does not know the future value of \( y \), but knows the type of the firm (i.e., he knows whether the income distribution is \( F^L(\cdot) \) or \( F^H(\cdot) \)). The supplier knows neither the future value of \( y \), nor the type of the firm he trades with. All he knows is a statement about the type of the firm, made by the manager prior to contracting the input. The message is represented by \( a \), with \( a \in \{h, l\} \), where \( h \) is the announcement for a \(H\)-type firm and \( l \) is the announcement for a \(L\)-type firm. Hence, the manager’s announcement strategy can be represented as a function \( s(i) : \{L, H\} \rightarrow \{l, h\} \) that defines for all types of firms the manager’s statement.

The manager of a \(L\)-type firm may honestly announce that the firm is of the \(L\)-type, or may lie and announce that the firm is of the \(H\)-type. In this model, the manager of the \(H\)-type firm would never lie.\(^4\) Therefore, the irrelevant action \( l = s(H) \) will be omitted in the following

\(^4\) The formal proof can be obtained by comparing the \(H\)-type firm manager’s payoffs in the two cases. Intuitively, the manager who declares that a good firm is bad would lose twice, since indirect costs go up and he may be fined for false statement.
developments.

Let $\Omega$ denote the supplier’s beliefs about the manager’s degree of honesty contingent on the type of firm. As $H$-firm managers never lie, suppliers assign a unit probability to the fact that a manager at the head of a high-return firm is honest. The subjective probability that the manager of a low-return firm is honest (announces that the firm is of type $L$) is denoted by $\mu$. For instance, if $\mu = 1$, the supplier believes that the manager is honest, if $\mu = 0$, the supplier believes that the manager is dishonest and if $\mu \in [0,1]$, the supplier believes that the manager randomizes between the two pure strategies with probability $\mu$ (alternatively, $\mu$ can be interpreted as the perceived frequency of honest managers in the total population of managers running $L$-type firms).

$$
\Omega = \begin{cases} 
\Pr[l|L] = \mu, & \text{where } \mu \in [0,1] \\
\Pr[h|H] = 1 
\end{cases}
$$

Given his beliefs and the observed signal, the supplier determines the offer price of the commodity unit according to a standard zero-profit condition. This price is posted immediately after the manager’s announcement about the type of the firm, and depends on this announcement. Hence the unit cost of the input is denoted by $c^a$ with $a \in \{l, h\}$. Then the trade takes place: the supplier delivers the input, but agrees on cashing the promised price at a later time, once that the firm realizes the output. In other words, the supplier grants the firm a trade credit.

At the end of the period, if the actual income of the firm is too small, the manager might not be able to fully comply with his liabilities, i.e. might not pay the full price; therefore, $c^a$ must include a premium related to the risk of payment default. The supplier is the residual claimant, hence, in the worst of cases (if the income is lower than the price agreed ex ante), he gets the firm’s income.

If the firm makes a positive profit ($y - c^a > 0$), the manager obtains a reward from work proportional to this profit (in order to keep the model as simple as possible, the manager gain is set equal to the profit).\footnote{Several recent studies put forward a strong relationship between manager compensation and a firm’s share value, probably explained by the dramatic increase in the share of option-based pay in total compensation over the nineties (Murphy, 1999; Hall, 2005).} If the firm defaults on its liabilities ($y < c^a$), the firm’s profit and the manager’s reward becomes zero.

\footnote{Several recent studies put forward a strong relationship between manager compensation and a firm’s share value, probably explained by the dramatic increase in the share of option-based pay in total compensation over the nineties (Murphy, 1999; Hall, 2005).}
According to the Sarbanes-Oxley Act, managers who disclose false information are subject to heavy sanctions (if caught). However, in practice, it is highly probable that if a fragile firm does not file for bankruptcy (because it benefits from a favorable income shock), inspectors cannot prove ex post that the manager has lied. To the contrary, when a firm gets bankrupt, it will fall under close public scrutiny. Hence, we assume in the following that an external inspection board checks files of all the bankrupt firms, and impose a fine $z$ (with $z > 0$) on managers who have delivered false statements.

Figure 1 represents the basic sequence of decisions:

At the outset of the game, Nature chooses the type $i$ of the firm; next, depending on the type of the firm, the manager makes his optimal statement, $a$. Then, given the signal issued by the manager, the supplier upgrades his prior beliefs and posts a price $c^a$ for the input. The dotted line linking the upper points indicates that the supplier who observes the signal $h$ does not know whether he trades with a $H$ or a $L$-type firm. Next step, Nature decides on the income of the firm, $y$. Depending on whether the firm’s resources suffice (or not) to pay the contracted price, the firm is either solvent or not solvent; in this latter case, the firm is pulled out of the market.
and the residual revenue is transferred to the supplier.

Remark that both $H$ and $L$-type firms may be subject to default. The manager’s payoff at the end of the game depends on the type of the firm, his announcement and Nature’s choice of output.

3 Equilibrium

A Bayesian Equilibrium of this game is defined as a pair $(s, \Omega)$ such that a manager’s announcement strategy $s$ maximizes his expected payoff given the supplier’s beliefs $\Omega$, and the supplier’s beliefs are correct given $s$. A separating configuration implies that the manager’s announcement unambiguously reveals the firm’s type $(s(H) = h$ and $s(L) = l)$. A pooling configuration appears when all managers deliver the same signal whatever the type of the firm; in this model, $s(i) = h$, $\forall i$. The game presents a Hybrid Bayesian Equilibrium (HBE), where some managers running low expected return firms will communicate truthfully, and some will lie; in this case, the equilibrium frequency of honest $L$-type firms’ managers ($\mu$) belongs to the interval $[0, 1]$. Pure strategy equilibria can then be interpreted as special cases of this hybrid equilibrium, which obtain for $\mu \to 0$ (the pooling case) and $\mu \to 1$ (the separating case).

The consequences from increasing the sanction $z$ on $\mu$, $c^h$ and the other relevant variables can be put forward as the outcome of explicit calculations when the income $y$ is uniformly distributed on $[0, \tau^i]$. In this particular case, the assumption $\bar{y}^H > \bar{y}^L$ is tantamount to $\tau^H > \tau^L$ ($\tau^i$ is the upper bound of the income distribution). Numerical solutions can be obtained for all distributions defined on positive supports, such as the lognormal one. In this case, $\tau^i \to \infty$ and $\bar{y}^H > \bar{y}^L \Leftrightarrow F^H(y) > F^L(y), \forall y$. In the following Section, we solve the model for the uniform distribution case; we will present in a special section the numerical simulations for a lognormal distribution.

To determine the equilibrium of the game, we first have to define the input price depending on the announcement. This price is needed in order to determine the objective probability of default which, in turn, has a bearing on the manager’s expected payoff. The objective probability of payment default of the $i-$type firm whose manager has announced $a$ is defined as $\Pr[y < c^a]$. 
3.1 The input price defined

As already mentioned, the input market is perfectly competitive, with free entry.

Let us now consider a supplier who observes the signal issued by the manager. Given his beliefs (Eq. 1), the probabilities he assigns to the type of the borrower contingent upon the signal issued by the manager, that is \( \Pr[i|a] \), are:

\[
\begin{align*}
\Pr[L|l] &= 1 \\
\Pr[H|h] &= \frac{\Pr[h|H]\Pr[H]}{\Pr[h|H]\Pr[H] + \Pr[h|L]\Pr[L]} = \frac{q}{1 - \mu(1 - q)}
\end{align*}
\]

We have denoted by \( c^a \) the price charged by the supplier, depending on the signal \( a \) issued by the manager. If the firm’s income is large enough \((y > c^a)\), the supplier will receive the full price \( c^a \). If the firm’s income is lower than the contracted price \((y < c^a)\), the supplier, who is the residual claimant, will get \( y^i \). We denote by \( c \) the cost for the supplier to produce the essential commodity unit (this cost is common knowledge).

6 To focus on the default premium, in the following we assume that suppliers, who act as trade lenders, are risk neutral individuals.

a) If the supplier receives the signal \( l \), he can unambiguously infer that he deals with a low-return firm (type \( L \)). Under the zero profit condition, the price \( c^l \) is implicitly defined by:

\[
c = \int_0^{c^l} ydF^L(y) + \int_{c^l}^\infty c^l dF^L(y).
\]

(3)

Considering an uniform distribution for the income of the \( L \)-type firm, \( c^l \) is the solution of:

\[
c = c^l - \frac{(c^l)^2}{2\tau^L},
\]

(4)

where \( c^l > c \). It can be checked that a price \( c^l \) exists if \( \tau^L > 2c \). Remark also that the solution \( c^l \) is independent of \( z \).

b) If the supplier gets the signal \( h \), he must take into account the possibility that the good signal might have been issued by a low return firm. Hence, the posted price \( c^h \) is implicitly defined by:

\[
c = \Pr[H|h]\left(\int_0^{c^h} ydF^H(y) + \int_{c^h}^{\tau^H} c^h dF^H(y)\right) + \Pr[L|h]\left(\int_0^{c^h} ydF^L(y) + \int_{c^h}^{\tau^L} c^h dF^L(y)\right)
\]

(5)

6 Alternatively, \( c \) might be seen as a certain price that the supplier might get in a risk-less trade.
or, given the uniform distribution of the firm’s income, by:

\[ c = \Pr[H|h] \left( -\frac{(c^h)^2}{2\tau^h} + c^h \right) + (1 - \Pr[H|h]) \left( -\frac{(c^b)^2}{2\tau^b} + c^b \right). \]  

(6)

In the next section, \( \Pr[H|h] \) appears to be a function of \( z \); hence the solution \( c^h \) depends on this variable.

Equations (4) and (6) could be solved to obtain explicit forms for \( c^l \) and \( c^h \). Yet calculations with roots of second degree equations are neither very aesthetic nor easily tractable. Such difficulties can be overcome if we use for former developments the difference between \( c^l \) and \( c^h \).

The difference must be positive, given the uncertainty related to dealing with a manager who announces \( h \) (there is no uncertainty related to announcing \( l \)). By substracting Eq. (6) from Eq (4), we get:

\[ c^l - c^h = \frac{(c^b)^2 \Pr[H|h] \left(1 - \frac{\tau^b}{\tau^L}\right)}{2\tau^L - (c^l + c^h)} > 0. \]  

(7)

In order to focus on a non trivial case, we admit that \( \tau^L > c^l \). If this condition does not hold, no supplier will accept to lend to a manager who declares that the firm is of the \( L - type \).

3.2 Conditions of existence of a HBE

Let us denote by \( W[a|L] \) the expected payoff of the manager at the head of the \( L\)-type firm who issues a signal \( a \). This payoff is related to the manager’s reward for profits (identical to profits if these are positive, zero if else), less the fine for fraudulent statements, to be charged only in the case of default. Formally, the expected payoff of the manager of a \( L - type \) firm who announces \( l \) is:

\[ W[l|L] = \int_{c^l}^{\tau^L} (y - c^l) dF^L(y) \]
\[ = \frac{1}{2} \tau^L - c^l \left(1 + \frac{\tau^b}{\tau^L}\right) + \frac{1}{2} (c^l)^2 \tau^L. \]

(8)

(9)

And the expected payoff of the manager who announces \( h \) is:

\[ W[h|L] = \int_{c^h}^{\tau^L} (y - c^h) dF^L(y) - \int_{0}^{c^h} dF^L(y) \]
\[ = \left[ \frac{1}{2} \tau^L - c^h \right] + \left( \frac{c^h}{\tau^L} \right) \left[ \frac{1}{2} c^h - z \right]. \]

(10)

(11)
A hybrid equilibrium exists if the $L$-type firm’s manager is indifferent between announcing $l$ or $h$:

$$ W[h|L] = W[l|L] $$

$$ \int_{c^h}^{\tau^L} (y - c^h) \, dF^L(y) - z \int_{0}^{c^h} dF^L(y) = \int_{c^l}^{\tau^L} (y - c^l) \, dF^L(y). \quad (12) $$

In the uniform distribution case, the former equation simplifies to:

$$ (c^l - c^h) [2\tau^L - (c^h + c^l)] = 2zc^h. \quad (13) $$

After replacing the term $(c^l - c^h)$ as defined by Eq. (7) into Eq.(13), the necessary condition for a HBE becomes:

$$ \Pr[H|h] = \frac{2z\tau^H}{(\tau^H - \tau^L)c^h}. \quad (14) $$

We can now be more specific about the nature of the equilibrium. Recall the definition of $\Pr[H|h]$ building on supplier’s beliefs (Eq. 2):

$$ \Pr[H|h] = \frac{q}{1 - \mu(1 - q)} \in [q, 1]. \quad (15) $$

Writing the equality between Eq. (14) and Eq. (15), the equilibrium frequency $\mu$ of honest $L$-type firms’ managers can be written:

$$ \mu = \frac{2z\tau^H - q(\tau^H - \tau^L) + c^h}{2z\tau^H(1 - q)} = G(z). \quad (16) $$

The hybrid equilibrium exists for $\mu \in [0, 1]$: this can happen if $z \in [z_1, z_2]$, with the two bounds implicitly defined by $G(z_1) = 0$ and $G(z_2) = 1$.

If $z \in [0, z_1]$, the strategy of honesty cannot be optimal for the manager of the $L$-type firm. The pooling equilibrium – where all managers announce that their firm is a high return one – occurs ($\mu = 0$). Hence, the antifraud policy would become effective only if the sanction exceeds a critical threshold, $z > z_1$.

If $z \in [z_2, \infty]$, the separating equilibrium emerges: all managers announce the true type of their firm, honesty is generalized ($\mu = 1$).

We focus hereafter on the hybrid equilibrium, which encompasses as special cases the pooling and separating situations. These explicit solutions are provided for the uniform distribution case.

---

7 In next subsection we will show that, in the hybrid equilibrium, the frequency of honest managers is an increasing function in the sanction level, $d\mu/dz > 0$ (Proposition 2). We also notice that since $c^h$ is solution to a second degree equation (Eq. 6), the explicit forms of $z_1$ and $z_2$ are not very aesthetic. They need not be displayed here.
4 Consequences of a tougher sanction

Consequences of a tougher sanction can be analyzed by studying the impact of $dz > 0$ on the main variables. Both the input price $c^h$ and the proportion of honest managers running $L$-type firms depend on the sanction. It turns out that:

**Proposition 1** In the hybrid equilibrium, the input price for managers who announce that their firm is of the $H$-type is decreasing with the sanction level.

**Proof.** We replace $\Pr[H|h]$ such as defined by Eq. (14) in Eq. (6):

$$c^h = c + \frac{(c^h)^2}{2\tau^H} \left( \frac{2z}{c^h} \right) \left( \frac{\tau^H}{\tau^H - \tau^L} \right) + \frac{(c^h)^2}{2\tau^L} \left[ 1 - \frac{2z}{c^h} \right] \left( \frac{\tau^H}{\tau^H - \tau^L} \right)$$

$$= c - \frac{2c^h}{\tau^L} + \frac{(c^h)^2}{2\tau^L}. \quad (17)$$

Differentiating the former expression, we get:

$$\frac{dc^h}{dz} = - \frac{c^h}{z + \tau^L - c^h} < 0 \quad (18)$$

**Proposition 2** In the hybrid equilibrium, the frequency of honest $L$-type firms’ managers ($\mu$) is increasing with the sanction level.

**Proof.** From Eq. (16), we obtain:

$$\frac{d\mu}{dz} = \left( \frac{q}{1-q} \right) \left( \frac{\tau^H - \tau^L}{2\tau^H} \right) \left( \frac{c^h}{z^2} \right) \left( \frac{2z + \tau^L - c^h}{z + \tau^L - c^h} \right) > 0. \quad (19)$$

These first two propositions are rather trivial. A higher sanction helps reducing dishonest disclosure by managers running $L$-type firms and, since the quality of the signal $H$ improves, the risk of doing business with a manager who declares that he runs a good company declines, hence the supplier may reduce the input price.

Time has come now to investigate the welfare implications of this measure. The concept of welfare is not easy to grasp within the framework of this simple model. However, two indicators may convey useful insights. Firstly, in this model, the default premium – defined as the cost in
excess over \( c \) that firms have to pay for the input – exactly measures the economic cost of expected default because suppliers, being risk neutral and perfectly competitive, price the firm as residual claimants who break even. This default premium has a real economic effect because requiring a higher default premium results in higher input prices, which in turn can bring about a self-fulfilling increase in the frequency of bankruptcy. Hence, minimizing the total default premium seems to be a reasonable welfare criterion. We can state:

**Proposition 3** In the hybrid equilibrium, the overall default premium is increasing with the sanction level.

**Proof.** We denote by \( \Gamma \) the overall price of the input as posted by suppliers at the beginning of the period. This price corresponds to the total amount of resources borrowed by firms from their input providers. We can define:

\[
\Gamma = q c^h + (1-q) \left[ \mu c^f + (1-\mu) c^h \right] \tag{20}
\]

The excess borrowing cost (as compared to the perfect information set-up) is \( \Gamma - c \). Its derivative with respect to the sanction can be written:

\[
\frac{d(\Gamma - c)}{dz} = \frac{d\Gamma}{dz} = [q + (1-q)(1-\mu)] \frac{dc^h}{dz} + (1-q) \left( c^f - c^h \right) \frac{dp}{dz} \tag{21}
\]

Replacing \( \frac{dc^h}{dz} \) and \( \frac{dp}{dz} \) by their expressions in Eq. (18) and Eq.(19), and \( (1-\mu) \) by its equilibrium value (Eq.16), we obtain:

\[
\frac{d\Gamma}{dz} = \frac{qc^h (\tau^H - \tau^L)}{2z^2 \tau^H [z + \tau^L - c^h]} \left[ -zc^h + (c^f - c^h) \left( 2z + \tau^L - c^h \right) \right] \tag{22}
\]

The sign of \( \frac{d\Gamma}{dz} \) is the same as the sign of the expression between braces. We show that:

\[
LHS(z) \equiv (c^f - c^h) \left( 2z + \tau^L - c^h \right) > zc^h \equiv RHS(z) \tag{23}
\]

Indeed, this is true for \( z = 0 \). Since

\[
\frac{dLHS(z)}{dz} - \frac{dRHS(z)}{dz} = \frac{\left( 2\tau^L - c^h \right) \left( c^f - c^h \right) + 2zc^f}{z + \tau^L - c^h} > 0, \tag{24}
\]

the inequality will be true whatever \( z > 0 \). Hence, \( d\Gamma/dz > 0 \).
When the sanction level increases, more managers will honestly announce the type of the firm \((\mu \text{ goes up})\), the value of the signal \(h\) increases leading to a lower price \(c^h\). But the number of managers who benefit of this better offer declines (there are less dishonest managers at the head of low return firms). At the same time, the number of managers that have to pay the bigger price \(c'\) increases. In this model, the average input price that managers have to pay is increasing with the sanction level.

Secondly, we follow Morris and Shin (2004) and consider that the number of defaulting firms may also be of interest. It can be surmised that bankruptcies come with substantial private losses imposed on workers and the other stakeholders, which do not appear in a firm’s book value. We show that:

**Proposition 4** *In the hybrid equilibrium, the proportion of firms that default on the loans from their input suppliers is increasing with the sanction level.*

**Proof.** Let \(\nu\) denote the frequency of defaulting firms in this economy. It depends on the distribution of firms between high and low return firms \((q, 1 - q)\), on the frequency of dishonest managers \((1 - \mu)\) in the population of \(L\)-type firms, and on the default rate recorded in each population of firms:

\[
\nu = qF^H (c^h) + (1 - q) [\mu F^L (c') + (1 - \mu) F^L (c^h)]
\]

\[
= \frac{q}{\tau^H} c^h + \frac{(1 - q)}{\tau^L} [\mu (c' - c^h) + c^h].
\]

(25)

Given that \(dc(\mu^L)/dz = 0,\)

\[
\frac{d\nu}{dz} = \frac{dc^h}{dz} \left[ \frac{q}{\tau^H} + \frac{(1 - q)(1 - \mu)}{\tau^L} \right] + \frac{d\mu}{dz} \frac{(1 - q)}{\tau^L} (c' - c^h).
\]

(26)

Replacing \(\frac{dc^h}{dz}\) and \(\frac{d\mu}{dz}\) by their expressions in Eq. (18) and Eq.(19), and \((1 - \mu)\) by its equilibrium value (Eq.16), we obtain:

\[
\frac{d\nu}{dz} = \frac{q c^h (\tau^H - \tau^L)}{2z^2 + \tau^L} \left[ (2z + \tau^L - c^h) (c' - c^h) - z (c^h - 2z) \right].
\]

(27)

This derivative is positive if the expression between braces is positive, that is if:

\[
(2z + \tau^L - c^h) (c' - c^h) > z (c^h - 2z).
\]

(28)

---

8 In our model, when a firm becomes bankrupt, creditors bear no additional costs related to the liquidation process. Indeed, the supplier (who is the residual claimant) is assumed to get the full residual income.
This is true, given that inequality (23), which is true, implies (28).

The rationale behind this less intuitive result follows the same logic as before (Proposition 3). Firstly, when the sanction increases, a higher proportion of $L$-type firms announce their type honestly. As a result, the pool of firms declaring to be $H$-type has a smaller proportion of firms that are actually $L$-type, and thus must pay a lower input price; in turn, this leads to a lower default rate on this price. However, the above effect is more than offset by the fact that a higher proportion of firms which are $L$-type now pays a promised input price that correctly reflects their risk of defaulting on it, and this leads to a higher default rate for these firms.

5 A numerical simulation with a lognormal distribution

In order to get some additional insight, we numerically solve the model in the case when the firm’s income follows a lognormal distribution. The lognormal distribution is well suited to this problem because the income can take only positive values. Such a distribution has no superior bound on the income ($\tau^1 \to \infty$). Hence, the assumption $\bar{y}^H > \bar{y}^L \Leftrightarrow F^H(y) > F^L(y), \forall y$.

For this example, we choose $F^L()$ such that the average income $\bar{y}^L = 1.60$ with a standard deviation $\sigma^L = 0.85$ and $F^H()$ such that $\bar{y}^H = 2.65$ with standard deviation $\sigma^H = 1.41$. We set the frequency of $H$-type firms to $q = 0.8$ and the cost of producing the essential commodity unit to $c = 1$.

The price charged by supplier for the input when the manager announces that the firm is of the $L$–type, i.e., $c^l$ is obtained as the solution to Eq. (3). In this example, $c^l = 1.078$.

The problem is then solved iteratively for various values of the sanction $z$. We allow $z$ to vary with a step of 0.001 in the closed interval from 0.168 to 0.205 so that $\mu$ is increasing from zero to one, covering the full range of hybrid equilibria. The hybrid equilibrium condition (Eq. 12) allows us to determine $c^h$. Finally, we solve Eq.(6) for $Pr[H|h]$, then obtain the equilibrium frequency of honest mangers at the head of $L$–type firms ($\mu$) from Eq. (2). More precisely, $\mu = \frac{Pr[H|h][q]}{Pr[H|h][1-q]}$ with $\mu \in [0,1]$ in the hybrid equilibrium.

When $z$ goes up, the cost $c^h$ declines from 1.019 to 1.008 (managers who announce that their firm is solid have to pay less and less). The frequency of defaults $\nu$ increases from 8.9% to 9.5%
(Figure 2), and the average cost of the input $\Gamma$ increases from 1.019 to 1.022 (Figure 3).

Taking into account the arc-elasticities, a ten percent increase in the sanction $z$ brings about a reduction in the cost $c^h$ by 4.75 percentage points, an increase in the frequency of defaults $\nu$ by 2.75 percentage points and an increase in the average cost of the input $\Gamma$ by 0.12 percentage points.

We solved the problem for several values of $y^i$, $\sigma^i$ and $c$, without being able to falsify the conclusion of the uniform distribution case. The Maple programme that allows to perform this simulation is given in the Appendix.

## 6 Conclusion

According to conventional wisdom in economics, more transparency is always better than less. This paper challenges to some extent this conjecture as applied to businesses’ communication policy. In our model, lenders have only imperfect information about the intrinsic quality of the borrower. They update their beliefs according to a signal issued by the manager himself, who has a perfect knowledge about the type of the firm. A honest manager will tell the truth about the
Figure 3: Average input cost as a function of $z$

type of the firm, a dishonest one would say that a fragile firm is actually solid. Transparency is here interpreted as the amount of separation between honest and dishonest managers, brought about by an increase in the sanction for fraudulent disclosure, such as implemented by the US in the aftermath of the corporate scandals of the late nineties.

Our analysis has focused on two welfare indicators, the average default premium and the frequency of defaulting firms. In this model with risk-neutral agents, the former indicator is a correct measure of the economic cost of expected default. We show that both the economy-wide default premium and the overall frequency of defaulting firms rise when the sanction for fraudulent disclosure is pushed up. This conclusion is not fully independent of our main assumptions. In particular, in order to obtain closed form solutions, we firstly considered that the firms' income distribution is uniform and then we performed numerical simulations with lognormal distributions. With another distribution, the opposite effects affecting fragile and pretended solid firms may entail a more ambiguous net effect. Yet the basic logic of the model holds whatever the income distribution.

In general, when more managers running fragile firms declare honestly the type of their company,
these firms will be submitted to additional financial pressure, which will in turn exacerbate the original difficulties. Whether the positive effect on good firm or the adverse effect on fragile firms dominates is ultimately a matter of empirical observation. What our analysis emphasizes it that fragile firms’ self-fulfilling troubles should not be underestimated by policymakers, and that an excessive sanction may bring about non-negligible economic effects.

7 References


A Detailed calculations

A.1 Determining the expression of \([c^l - c^h]\)

To determine \(c^l\), we write the zero trade-off condition:

\[
c = \int_0^{c^l} ydF^L(y) + \int_{c^l}^{\tau_L} c^ldF^L(y)
= c^l + \int_0^{c^l} (y - c^l)dF^L(y)
= c^l + \frac{1}{\tau_L} \left[y\left(\frac{1}{2}y - c^l\right)\right]_0^{c^l}
= c^l + \frac{c^l}{\tau_L} \left(\frac{1}{2}c^l - c^l\right)
= c^l - \frac{(c^l)^2}{2\tau_L}
\]

(A.29)

We get:

\[
c^l = c + \frac{(c^l)^2}{2\tau_L}
\]

In the same way, \(c^h\) results form the zero trade-off condition:

\[
c = \Pr[H|h]\left(\int_0^{c^h} ydF^H(y) + \int_{c^h}^{\tau_H} c^hdF^H(y)\right) + \Pr[L|h]\left(\int_0^{c^h} ydF^L(y) + \int_{c^h}^{\tau_L} c^hdF^L(y)\right)
= \Pr[H|h]\left(\int_0^{c^h} \frac{y^2}{2\tau_H} dy + c^h\int_{c^h}^{\tau_H} \frac{1}{\tau_H} dy\right) + \Pr[L|h]\left(\int_0^{c^h} \frac{y^2}{2\tau_L} dy + c^h\int_{c^h}^{\tau_L} \frac{1}{\tau_L} dy\right)
= \Pr[H|h]\left(\frac{(c^h)^2}{2\tau_H} + c^h \left[\frac{y}{\tau_H}\right]_{c^h}^{\tau_H}\right) + \Pr[L|h]\left(\frac{(c^h)^2}{2\tau_L} + c^h \left[\frac{y}{\tau_L}\right]_{c^h}^{\tau_L}\right)
= \Pr[H|h]\left(-\frac{(c^h)^2}{2\tau_H} + c^h\right) + \Pr[L|h]\left(-\frac{(c^h)^2}{2\tau_L} + c^h\right)
\]

(A.30)

We get:

\[
c^h = c + \Pr[H|\tau^H]\frac{(c^h)^2}{2\tau_H} + \Pr[L|\tau^L]\frac{(c^h)^2}{2\tau_L}
\]

The difference is:

\[
c^l - c^h = c + \frac{(c^l)^2}{2\tau_L} - c - \Pr[H|h]\frac{(c^h)^2}{2\tau_H} - \Pr[L|h]\frac{(c^h)^2}{2\tau_L}
= \frac{(c^l)^2}{2\tau_L} - \Pr[H|h]\frac{(c^h)^2}{2\tau_H} - (1 - \Pr[L|h])\frac{(c^h)^2}{2\tau_L}
= \frac{(c^l)^2 - (c^h)^2}{2\tau_L} - \Pr[H|h]\left[\frac{(c^h)^2}{2\tau_H} - \frac{(c^h)^2}{2\tau_L}\right]
= \frac{(c^l + c^h)}{2\tau_L} \left((c^l - c^h) - \Pr[H|h](c^h)^2 \left(\frac{1}{2\tau_H} - \frac{1}{2\tau_L}\right)\right)
\]
or, in an equivalent way:

\[
(c^l - c^h) \left[ 1 - \frac{c^l + c^h}{2\tau^L} \right] = \Pr[H|h](c^h)^2 \frac{(\tau^H - \tau^L)}{2\tau^L \tau^H} \\
(c^l - c^h) \left[ 2\tau^L - (c^l + c^h) \right] = \Pr[H|h](c^h)^2 \frac{(\tau^H - \tau^L)}{\tau^H}
\]

We obtain

\[
(c^l - c^h) = \frac{(c^h)^2 \Pr[H|h] \left( 1 - \frac{\tau^L}{2\tau^H} \right)}{2\tau^L - (c^l + c^h)}.
\]

### A.2 Defining the HBE

The expected payoff of the \(L\)-type firm manager who fairly announces \(\tau^L\),

\[
W[\hat{\tau}^L|\tau^L] = \int_{c^l}^{\tau^L} (y - c^l) \, dF^L(y) \\
= \frac{1}{\tau^L} \left[ y \left( \frac{1}{2}y - c^l \right) \right]_{c^l}^{\tau^L} \\
= \frac{1}{\tau^L} \left[ \tau^L \left( \frac{1}{2} \tau^L - c^l \right) - c^l \left( \frac{1}{2}c^l - c^l \right) \right] \\
= \frac{1}{\tau^L} \left[ \tau^L \left( \frac{1}{2} \tau^L - c^l \right) - c^l \left( -\frac{1}{2}c^l \right) \right] \\
= \left( \frac{1}{2} \tau^L - c^l \right) + \frac{1}{2} \left( \frac{c^l}{1} \right)^2 
\]  

(A.31)

The expected payoff of the manager who announces \(\hat{\tau}^H\) (the dishonest one) is:

\[
W[\hat{\tau}^H|\tau^L] = \int_{c^h}^{\tau^L} (y - c^h) \, dF^L(y) - \int_0^{c^h} dF^L(y) \\
= \frac{1}{\tau^L} \left[ y \left( \frac{1}{2}y - c^h \right) \right]_{c^h}^{\tau^L} - \frac{c^h}{\tau^L} \\
= \frac{1}{\tau^L} \left[ \tau^L \left( \frac{1}{2} \tau^L - c^h \right) \right] - \frac{1}{\tau^L} \left[ c^h \left( \frac{1}{2}c^h - c^h \right) \right] - \frac{c^h}{\tau^L} \\
= \left( \frac{1}{2} \tau^L - c^h \right) + \frac{c^h}{\tau^L} \left( \frac{1}{2}c^h - \frac{1}{2}c^h \right)
\]  

(A.32)

The HBE condition can be written:

\[
W[h|L] = W[l|L] \\
\left( \frac{1}{2} \tau^L - c^h \right) - \frac{1}{\tau^L} c^h \left( -\frac{1}{2}c^h + z \right) = \frac{1}{\tau^L} \left[ \tau^L \left( \frac{1}{2} \tau^L - c^l \right) + \frac{1}{2} \left( c^l \right)^2 \right] \\
(c^l - c^h) \tau^L + \frac{1}{2} (c^h - c^l) (c^h + c^l) = z c^h \\
(c^l - c^h) 2\tau^L + (c^h - c^l) (c^h + c^l) = 2zc^h \\
(c^l - c^h) \left[ 2\tau^L - (c^l + c^h) \right] = 2zc^h
\]
But we know that:
\[ c' - c^h = \frac{(c^h)^2 \Pr[H|\tau^H]}{2\tau^L - (c' + c^h)} \]

thus the HBE condition becomes:
\[ 2zc^h = (c^h)^2 \Pr[H|h] \left[ 1 - \frac{\tau^L}{\tau^H} \right] \]

\[ \Leftrightarrow \Pr[H|h] = \frac{2z\tau^H}{(\tau^H - \tau^L)c^h} \]

which is Condition 14 in the text.

A.3 Consequences of a tougher sanction

We start from Eq. 25:
\[ \nu = q \frac{c^h}{\tau^H} + \frac{(1 - q)}{\tau^L} \left[ \mu (c' - c^h) + c^h \right] \]

We know that \( \frac{d\nu}{dz} = 0 \). The derivative of the frequency of defaults with respect to \( z \) can be written:

\[ \frac{d\nu}{dz} = \frac{q}{\tau^H} \frac{dc^h}{dz} + \frac{(1 - q)}{\tau^L} \left\{ \frac{d\mu}{dz} \left( c' - c^h \right) - \mu \frac{dc^h}{dz} + \frac{dc^h}{dz} \right\} \]

\[ = \frac{dc^h}{dz} \left[ \frac{q}{\tau^H} + \frac{(1 - \mu)(1 - q)}{\tau^L} \right] + \frac{d\mu}{dz} \frac{(1 - q)}{\tau^L} (c' - c^h) \]

**Proposition 1.** Calculus of \( \frac{dc^h}{dz} \). We replace \( \Pr[H|h] \) such as defined by Eq. 14 in Eq. 6:

\[ c^h = c + \frac{2z\tau^H}{(\tau^H - \tau^L)c^h} \frac{(c^h)^2}{2\tau^H} \left[ 1 - \frac{2z\tau^H}{(\tau^H - \tau^L)c^h} \right] \frac{(c^h)^2}{2\tau^L} \]

\[ = c + \frac{2z\tau^H}{(\tau^H - \tau^L)c^h} \frac{c^h}{2\tau^H} - \frac{2z\tau^H}{(\tau^H - \tau^L)c^h} \frac{c^h}{2\tau^L} + \frac{(c^h)^2}{2\tau^L} \]

\[ = c + \frac{zc^h}{(\tau^H - \tau^L)c^h} \left[ 1 - \frac{\tau^H}{\tau^L} \right] + \frac{(c^h)^2}{2\tau^L} \]

\[ = c - \frac{z}{\tau^L} c^h + \frac{1}{2\tau^L}(c^h)^2 \]

Differentiating the former expression with respect to \( z \) and \( c^h \):

\[ \frac{dc^h}{dz} = -\frac{c^h}{\tau^H} + \frac{z}{\tau^L} dc^h + \frac{c^h}{\tau^L} dc^h \]

\[ dc^h[1 + \frac{z}{\tau^L} - \frac{c^h}{\tau^L}] = -\frac{c^h}{\tau^L} dz \]

\[ dc^h[\tau^L + z - c^h] = -c^h dz \]

we get Eq.18 in the text:

\[ \frac{dc^h}{dz} = -\frac{c^h}{z + \tau^L - c^h} < 0 \]
Proposition 2. Calculus of \( \frac{d\mu}{d\tau} \). According to Eq. 16:

\[
\mu = \frac{2z\tau^H - qch (\tau^H - \tau^L)}{2z\tau^H (1 - q)} = \frac{1}{1 - q} - \frac{q}{2\tau^H} \left( \tau^H - \tau^L \right) c^h z
\]

we get Eq. 19 in the text:

\[
\frac{d\mu}{d\tau} = -\frac{q}{(1 - q)} \left( \frac{\tau^H - \tau^L}{2\tau^H} \right) \frac{z dc^h h}{d\tau} - c^h
\]

\[
= -\frac{q}{(1 - q)} \left( \frac{\tau^H - \tau^L}{2\tau^H} \right) \frac{z dc^h h}{d\tau} - c^h
\]

\[
= \frac{q}{(1 - q)} \left( \frac{\tau^H - \tau^L}{2\tau^H} \right) \left( \frac{c^h}{z^2} \right) \left( 2z + \tau^L - c^h \right)
\]

Also notice that:

\[
1 - \mu = \frac{q}{(1 - q)} \left( 1 - \tau^H - \tau^L \right) 2z\tau^H
\]

(33)

Proposition 3: The price of risk

We denote the global cost of the input by:

\[\Gamma = qc^h + (1 - q) [\mu c^l + (1 - \mu)c^h]\]

\[
\frac{d\Gamma}{d\tau} = q \frac{dc^h}{d\tau} + (1 - q) \frac{d [\mu c^l + (1 - \mu)c^h]}{d\tau}
\]

\[
= q \frac{dc^h}{d\tau} + (1 - q) \frac{d\mu}{d\tau} c^l + (1 - \mu) \frac{dc^h}{d\tau} c^l - c^h \frac{d\mu}{d\tau}
\]

We replace the derivatives in the last equation, to get:

\[
\frac{d\Gamma}{d\tau} = \left[ q + (1 - q) \frac{c^h (\tau^H - \tau^L) - 2z\tau^H}{2z\tau^H} - \frac{c^h}{z + \tau^L - c^h} \right] \left( -\frac{c^h}{z + \tau^L - c^h} \right)
\]

\[
+ (1 - q) (c^l - c^h) \frac{q}{(1 - q)} \left( \frac{\tau^H - \tau^L}{2\tau^H} \right) \left( \frac{c^h}{z^2} \right) \left( 2z + \tau^L - c^h \right)
\]

\[
= -q \frac{c^h (\tau^H - \tau^L)}{2z\tau^H} \left( \frac{c^h}{z + \tau^L - c^h} \right) + (c^l - c^h) q \frac{(\tau^H - \tau^L)}{2\tau^H} \left( \frac{c^h}{z^2} \right) \left( 2z + \tau^L - c^h \right)
\]

\[
= q \frac{(\tau^H - \tau^L)c^h}{2z\tau^H (z + \tau^L - c^h)} \left\{ -z c^h + (c^l - c^h) (2z + \tau^L - c^h) \right\}
\]

We study the sign of the term between brackets:

\[\text{LHS}(z) \equiv (c^l - c^h) (2z + \tau^L - c^h) > z c^h \equiv \text{RHS}(z)\]
This is true for $z = 0$. We calculate the derivatives:

\[
\frac{dLHS(z)}{dz} = \frac{d}{dz} \left[ (2z + \tau L - c^h) \left(c^h - c^l\right) \right] = \frac{2 + \frac{c^h}{z + \tau L - c^h} \left(c^h - c^l\right) + (2z + \tau L - c^h) \frac{c^h}{z + \tau L - c^h}}{(2z + 2\tau L - c^h) \left(c^h - c^l\right) + (2z + \tau L - c^h) c^h} \tag{A.34}
\]

\[
\frac{dRHS(z)}{dz} = c^h + z \frac{dc^h}{dz} = c^h + z \left(-\frac{c^h}{z + \tau L - c^h}\right) \tag{A.35}
\]

\[
\frac{dLHS(z)}{dz} > \frac{dRHS(z)}{dz} \Rightarrow \frac{(2z + 2\tau L - c^h) \left(c^h - c^l\right) + (2z + \tau L - c^h) c^h}{z + \tau L - c^h} \frac{c^h}{z + \tau L - c^h} > \frac{c^h}{z + \tau L - c^h} \left(1 - \frac{z}{z + \tau L - c^h}\right) > c^h \left(\frac{\tau L - c^h}{z + \tau L - c^h}\right)
\]

\[
\frac{2 + \frac{c^h}{z + \tau L - c^h} \left(c^h - c^l\right) + (2z + \tau L - c^h) \frac{c^h}{z + \tau L - c^h}}{(2z + 2\tau L - c^h) \left(c^h - c^l\right) + (2z + \tau L - c^h) c^h} > 0
\]

\[
\frac{2z + 2\tau L - c^h) \left(c^h - c^l\right) + 2zc^h}{z + \tau L - c^h} > 0
\]

\[
\frac{(2\tau L - c^h) \left(c^h - c^l\right) + 2zc^l}{z + \tau L - c^h} > 0
\]

**Proposition 4.** The default rate
Thus \( \frac{dv}{dz} > 0 \) if:

\[
\text{LHS}(z) \equiv (2z + \tau^L - c^L) (c^L - c^h) > z (c^h - 2z) \equiv \text{RHS}(z)
\]  \hspace{1cm} (37)

ALTERNATIVE PROOF (from text):

We first notice that for \( z = 0 \), this condition is fulfilled.

\[
\text{LHS}(0) > \text{RHS}(0).
\]  \hspace{1cm} (38)

We check then that \( \frac{d\text{LHS}(z)}{dz} > \frac{d\text{RHS}(z)}{dz} \). If this is true, former condition holds whatever \( z > 0 \).

\[
\frac{d\text{LHS}(z)}{dz} = d \left[ (2z + \tau^L - c^h) (c^L - c^h) \right]
= \left( 2z + \frac{c^h}{z + \tau^L - c^h} \right) (c^L - c^h) + (2z + \tau^L - c^h) \frac{c^h}{z + \tau^L - c^h}
= \left( 2z + 2\tau^L - c^h \right) (c^L - c^h) + (2z + \tau^L - c^h) c^h
\]  \hspace{1cm} (A.39)

\[
\frac{d\text{RHS}(z)}{dz} = d \left[ z (c^h - 2z) \right]
= (c^h - 2z) - z \left( \frac{c^h}{z + \tau^L - c^h} + 2 \right)
= \left( z + \tau^L - c^h \right) (c^h - 2z) - z (2z + 2\tau^L - c^h)
\]  \hspace{1cm} (A.40)

\[
\frac{d\text{LHS}(z)}{dz} > \frac{d\text{RHS}(z)}{dz}
\]
\[
\frac{(2z + 2\tau^L - c^h)(c^l - c^h)}{z + \tau^L - c^h} + (2z + \tau^L - c^h)c^h > \frac{(z + \tau^L - c^h)(c^h - 2z)}{z + \tau^L - c^h} - z \frac{(2z + 2\tau^L - c^h)}{z + \tau^L - c^h}
\]

\[
(2z + 2\tau^L - c^h)(c^l - c^h) + (2z + \tau^L - c^h)c^h > (z + \tau^L - c^h)(c^h - 2z) - z^2 (z + 2\tau^L - c^h)
\]

\[
(2z + 2\tau^L - c^h)(c^l - c^h + z) + (2z + \tau^L - c^h)c^h > (z + \tau^L - c^h)(c^h - 2z)
\]

\[
(2z + 2\tau^L - c^h)(c^l - c^h + z) + z c^h + 2z (c^h + \tau^L - c^h) > 0
\]

\[
(2z + 2\tau^L - c^h)(c^l - c^h + z) + (2z + 2\tau^L - c^h) > 0
\]

\[
(2z + 2\tau^L - c^h)(c^l - c^h) > 0
\]  \hspace{1cm} (A.41)

\section*{B Maple programme - numerical simulation with lognormal distributions}

The two income distributions : notation : FL is X and FH is Z.

\begin{verbatim}
> with(Statistics);
> X := RandomVariable(LogNormal(.35, .5));
> PDF(X, u);
> Z := RandomVariable(LogNormal(.85, .5));
> PDF(Z, u);
> Mean(X); 1.608014197
> Mean(Z); 2.651167211
\end{verbatim}

The input cost for the firm that announces 'l' ; 'cl' as a solution to Equation (3).

\begin{verbatim}
> Eq1 := int(u*PDF(X, u), u = 0 .. cl)+int(cl*PDF(X, u), u = cl .. infinity)-c;
> c := 1;
> cl1 := fsolve(Eq1 = 0, cl);
1.077603183
\end{verbatim}

Parameter: The input cost without uncertainty

\begin{verbatim}
> c := 1;
> cl1 := fsolve(Eq1 = 0, cl);
1.077603183
\end{verbatim}
The input cost for the L firm that announces ‘h’ (in the hybrid equilibrium) - determines \( \Pr[H|h] = p \); Eq(5).

\[
\text{Eq2} := -c + p \cdot (\int_0^{\text{ch1}} u \cdot \text{PDF}(Z, u) \, du + \int_{\text{ch1}}^{\infty} \text{PDF}(Z, u) \, du) + (1-p) \cdot (\int_0^{\text{ch1}} u \cdot \text{PDF}(X, u) \, du + \int_{\text{ch1}}^{\infty} \text{PDF}(X, u) \, du);
\]

The hybrid equilibrium condition \( W[l|L] = W[h|L] \); here it determines ‘ch’ Eq(13).

\[
\text{Eq3} := \int_{\text{cl1}}^{\infty} (u - \text{cl1}) \cdot \text{PDF}(X, u) \, du - \int_{\text{ch}}^{\infty} (u - \text{ch}) \cdot \text{PDF}(X, u) \, du + z \cdot \int_0^{\text{ch}} \text{PDF}(X, u) \, du;
\]

Parameter: the frequency of good firms (H-type)

\[
q := 0.8;
\]

The frequency of honest L type managers (who announce L) ; \( \mu \) varies between [0,1] in the hybrid.

\[
\mu := \frac{p1 - q}{p1 \cdot (1 - q)};
\]

The sequence of defaulting firms \( v \):

\[
\mu := q \cdot \text{CDF}(Z, \text{ch1}) + (1-q) \cdot (\mu \cdot \text{CDF}(X, \text{cl1}) + (1-\mu) \cdot \text{CDF}(X, \text{ch1}));
\]

The average cost of the input:

\[
G := q \cdot \text{ch1} + (1-q) \cdot (\mu \cdot \text{cl1} + (1-\mu) \cdot \text{ch1});
\]

Plot & loop instructions

\[
z1 := 0.168;
\]

\[
z2 := 0.205;
\]

\[
\text{step} := 0.1e-2;
\]

\[
\text{nbz} := \text{round}((z2-z1)/\text{step}+1);
\]

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\[
\text{for } z \text{ from } z1 \text{ by step to } z2 \text{ do i := round((z-z1)/step+1); ch1 := fsolve(Eq3 = 0, ch); p1 := fsolve(Eq2 = 0, p); lz[i] := z; lch[i] := ch1; lp[i] := p1; lmu[i] := mu; lG[i] := G; lnu[i] := evalf(nu) end do;
\]

Main variables defined

Cost of the sanction, \( z \) (exogenous)

\[
\text{print(lz)};
\]
Cost ch

> print(lch);

Frequency : Pr[H|h]=p

> print(lp);

Frequency of honest L-managers : μ

> print(lmu);

Frequency of defaults in the economy : ν

> print(lnu);

c+Cost of risk Γ

> print(lG);

Couples of variables and plots

> ral := seq(i, i = 1 .. nbz);
> relzmu2 := [seq([lz[i], lmu[i]], i = ral)];
> relzG2 := [seq([lz[i], lG[i]], i = ral)];
> relznu2 := [seq([lz[i], lnu[i]], i = ral)];
> with(plots);
> pointplot(relznu2, style = point, axes = boxed, labels = [sanction, ‘.’(freq, defaults)]);
> pointplot(relzG2, style = point, labels = [sanction, Typesetting[delayDotProduct]('.’(av, input), cost, true)], axes = boxed);
> pointplot(relzmu2, style = point, labels = [sanction, Typesetting[delayDotProduct]('.’(fr, honest), managers, true)], axes = boxed);

Some important statistics (arc-elasticities)

> elazG := (lG[nbz]-lG[1])*lz[1]/(lG[1]*(lz[nbz]-lz[1]));
0.01180310198

> elaznu := (lnu[nbz]-lnu[1])*lz[1]/(lnu[1]*(lz[nbz]-lz[1]));
0.2756125041

> elazch := (lch[nbz]-lch[1])*lz[1]/(lch[1]*(lz[nbz]-lz[1]));
-0.04766121400
> restart;