Efficient tuition fees and examinations: A reply

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Abstract

Gary-Bobo and Trannoy (2008) assume that students observe a private noisy signal of their ability while university gets information through noisy tests. They derive that a selection of potential applicants mixing tuition fees and examinations is socially optimal when the information owned by the university is not completely revealed to students. If tests results are public knowledge, they deduce that tuition fees alone can lead to an optimal degree of student self-selection. In this article, we propose to consider a more realistic assumption, taking into account the heterogeneity of the population. Indeed, due to this heterogeneity, a part of the population has a biased perception of its own skills, leading to an underestimation of the capabilities to succeed in a higher education system. We show that this new assumption may lead to rather different conclusions. In particular, we derive that optimal tuition fees are lower than those obtained by Gary-Bobo and Trannoy (2008) for an homogeneous population, given the amount of students.

Introduction

In a recent article, Gary-Bobo and Trannoy (2008) support the idea that tuition fees can efficiently select students applying to the university if a set of conditions is fulfilled. The most important conditions are basically:

- the existence of imperfect and asymmetric information (students observe a private noisy signal on their ability and the university gets noisy tests concerning students’ abilities);
• the absence of student-loan imperfections in order to “guarantee” that good
even students coming from low income families will be able to study by
borrowing the necessary amount of money (which will be reimbursed once
working).

Imperfection and asymmetric information would lead to adverse selection. This
could be prevented or limited by high enough tuition fees. Indeed, they would
discourage the “too bad” students to apply for university studies\(^1\). In their
model, the level of tuition should be particularly high since university’s inform-
ation about students’ ability is low. Tuition fees would even constitute the best
way to select students under “one-sided asymmetric information” (i.e. if students
private information is not shared by the university while information obtained
by the university - through examination - is common knowledge)\(^2\).

Relying on this work\(^3\), the authors have proposed a more exhaustive and
normative article (Gary-Bobo and Trannoy (2005b)) that led them to identify
what could be a good way to select students in higher education (and secondarily
to finance educational institutions).

\(^1\)“Too bad” students are those whose ability is so low that the cost of higher education is
higher than the expected gains by means of education (i.e. higher wages net from university
costs).

\(^2\)Indeed, in this case, applicants know and use all the information that the university owns
about them. Since gains in terms of wage due to education increase both individual and social
welfare in the same way, it is then socially optimal to let potential applicants decide whether
apply or not, given the optimal tuition fees established in the model.

\(^3\)Before publication in the Journal, this research has led to the publication of a working
The stakes related to this research, as well as to many other works leading to similar conclusions, are particularly crucial for many reasons: Indeed, following the normative conclusions would mean to deeply transform the economic, social, cultural and institutional basis of the current educational system which will have a series of consequences that have to be further identified and analysed. It is thus primordial to check if the assumptions and the results of this research are robust before political implementation.

In this article we contribute to the debate and argue that Gary-Bobo and Trannoy (2008)’s model is sensitive to the change of a key and questionable assumption: The one of modelling individual’s talents in skilled activities by a simple random variable under the hypothesis of an homogeneous population. Then, relying on theoretical and empirical literature, we discuss more widely the limits of using the results obtained in their article for normative purposes.

The present article is organised as follows: Section 1 discusses two of the main assumptions of Gary-Bobo and Trannoy (2008)’s model and justifies to change one of them. Section 2 shows that their results are not robust with regards to this change. Finally, Section 3 widens this fundamental discussion about tuition fees and identifies perspectives for further research.
1 A critical analysis of Gary-Bobo and Trannoy (2008)’s main assumptions

1.1 Imperfect and asymmetric information

The first and main assumption that must be discussed is the one concerning the imperfect and asymmetric perception that individuals have about about their own talent.

In Gary-Bobo and Trannoy (2008)’s model, each individual has a “noisy” information about his ability. It is also the case of the university that tries to know about the applicant ability through examinations and qualitative assessments. As a result, the university also owns a noisy information about the applicants. Furthermore, an asymmetric information between applicant and university is modeled. It can be a “bilateral” one if the university does not know the information the student owns about its ability and when the student knows imperfectly the information the university got about him. It can be a “one-sided” asymmetry when university’s information is a common knowledge.

These types of asymmetries are quite realistic and can explain opportunistic behaviours of university applicants. In the model, these behaviours can be efficiently contradicted by implementing jointly tuition fees and examinations. Tuition fees appear to be particularly important in the selection process since the applicant owns an accurate information about his talent (as self-selection improves), since the university has difficulties to assess student’s ability (examinations and tuition fees become less substitutable) and since there is only
“one-sided” information asymmetry (university’s information is common knowledge)\(^4\).

However, this assumption does not take into account the fact that students’ behaviours also depend on their social environment. The importance of this factor has been highlighted in the literature, so that individuals having the same basic talent have different ability to fit with the social codes of examinations and a biased perception of their own ability: Students coming from the socially or culturally privileged classes tend to estimate more precisely (or to overestimate) their talent while students coming from disadvantaged families underestimate their own one\(^5\).

According to the seminal work of Bourdieu (1974), “Adolescents will behave [...] in order to achieve what he perceives as a fact: when one belongs to a disadvantaged background, we can not get into university. [...] The skill required in order to ‘choose’ the best objective strategies (e.g. selecting a financial investment, a school or a career) is very unequally distributed. It varies almost exactly like the power of which the success of these strategies depends on. [...] Thus, even at a high level of curriculum and despite the effects of over-selection, we find that students are particularly modest in their academic ambitions (as well as in the assessment of their results) and particularly limited in their career

\(^4\)See footnote n°2.

\(^5\)The bias regarding the ability to efficiently take an examination in relation to the social or cultural environment will not be addressed in the model presented in the Section 2. This article will be limited to the bias regarding individual’s perception of their talent. However, those two effects reinforce each other when talking about our bellow results.
projects because they belong to groups whose educational opportunities are the lowest. " (p. 6, 8 and 9).

These "bad academic investment" could particularly be explained by asymmetric information about educational curriculum courses of the different classes of agents\(^6\), by the lack of alternative opportunities in case of failure for individuals not enjoying big enough social capital, or by their lack of familiarity with the positions that could be reached through education. The investment into education also reflects a desire from privileged classes to maximise "symbolic" returns, beyond the maximisation of economic ones\(^7\).

Several recent empirical studies confirm the strong correlation between the social characteristics of an individual and its academic perspectives. This is consistent with the hypothesis of biased "information" contingent on the various social groups about their ability to "succeed" in studies: Finnie et al. (2005), show on Canadian data that family characteristics (level of parental education, family type, ethnicity, place of residence) have significant effects on university registration. There is a rich literature on the above mentioned issues and consistent work demonstrating that "Children who grow up in a poor or low income family tend to have lower educational and labour market attainments than children from more affluent families [...]]" (see Haveman and Wolfe (1995), p.1870).

\(^6\)According to Bourdieu (1974), p.13, "This gap can also lead to inappropriate strategies, because they are fulfilled out of time: employees whose careers have been limited since they did not pass the baccalauréat [A-levels] often extend their investments until their children have this degree but only until this, even though this degree no longer meets the negative and positive functions it formerly filled [...]]".

In the 1970s, the results of a seminal research have shown that until "one third of the measured role of education in attainments reflects the influence of family background [...]" (p.1841). More recent studies using more accurate in-depth empirical techniques estimate a much higher link. As Haveman and Wolfe (1995) pointed out, sociological and economical results are consistent on this issue even if they rely on rather different theoretical and methodological backgrounds. They also mentioned that the same goes for work in psychology which emphasises the role of some stressful family events (divorce, parental unemployment...) and of parents' psychological resources.

For all these reasons, the assumption by Gary-Boo and Trannoy (2008), according to which the quality of individuals' information about their own talents would be homogeneous among the population, appears very questionable.

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8 "All of these studies find correlations approximately twice as high as those of the earlier studies, in part as a result of the errors in variables and life-cycle problems affecting the earlier studies. Their findings call into question Becker's conclusion in 1988 that "low earnings as well as high earnings are not strongly transmitted from fathers to sons" (p. 10)" (Haveman and Wolfe (1995), p.1843).

9 "The Socialisation/Role Model Perspective [...] stresses the potentially important effect of role models and socialisation (adults or peers to whom children or adolescents relate and who set norms of behaviour and achievement to which they aspire) during childhood and adolescent years on achievements as young adults. [...] While the channels of transmission in this framework are quite different from those emphasised by economists, the implications of this perspective are consistent with the economist's with respect to the potential effects of parental education, labour supply, and fertility choices on children's attainments" (Haveman and Wolfe (1995), p.1834-1835).
1.2 Borrowing constraints

A second assumption of Gary-Bobo and Trannoy (2008), stating that there is no borrowing constraint, should also be debated. Indeed, far from being independent of information perception, this discussion will reinforce the point we developed in the previous section.

This assumption is widely used in the literature in order to avoid the problem of education funding (short-term constraint) that arises for "talented" students coming from the working class. It implies that a loan system (usually without or low-interest rate) is available to all and opens “pay-as-you-go” reimbursement possibility once the individual has entered the professional life.

Nevertheless, this approach has a strong limitation since it considers that it is equivalent, in the decision of studies continuation, to come from a family which finances educational costs and to benefit from the “right” to borrow for this purpose. Indeed, in the model of Gary-Bobo and Trannoy (2008), the decision to attend university depends only on the difference between the expected wage rate gain earned by means of higher education (net of tuition fees) and the unskilled worker’s wage rate. In the model, this difference does not depend on the social environment of the student but on is basic talent (randomly distributed within the population - assumption that we will keep).

The reality and the strength of this borrowing constraint are discussed in the literature, in relation to other key variables. Relying on Canadian data, Frenette have found a small importance of the borrowing constraint: Only 12% of the gap in university attendance between students whose parents are in the
highest income quartile and those whose parents belong to the lowest quartile was related to financial constraints (Frenette (2007)). The parents educational level, their expectations and the quality of secondary school have even more important explanatory power. These results are also consistent with those of Carneiro and Heckman (2002) or Keane and Wolpin (2001), the latter highlighting, through a counter-factual methodology, that the elimination of the (still strong) borrowing constraints would not induce significant effect on university enrolment. Indeed, university registration and academic success would mainly be explained by other types of transfers between parents and children (such as tutoring) than financial constraints.

Contrary to this perspective “minimising” financial constraints, other articles insist on the impact of the debt burden on the decision to apply to the university, to choose the educational field and to favour a type of occupation (in public administration or private sector, for example). Performing empirical and experimental studies on students, Field (2009) shows that even a perfect access to credit may be insufficient to avoid distortions related to debt burden: if the borrowing behaviours and the reactions to debt burden are not rational then career choices will be distorted in favour of lucrative jobs to the detriment of jobs that could be socially useful.

The differences between tuition fees - in terms level and growth (see Cameron and Heckman (2001)) - have an impact on the choice of the university (particularly between the public and private ones). In addition, certain other evidences show that although borrowing constraints are usually low, some social groups
may be more sensitive than others. Following Plug and Vijverberg (2005): “high-ability children in low-income families face binding credit constraints that society may wish to relieve” (p.1). This is also the view of Frenette (2005) who demonstrates the highly negative impact of the unexpected increase of tuition fees on the Ontario middle class chance to continue studies.

Similarly, relying on the British case analysis, Callender (2006) argues that “evidence suggests that debt deters university entry among certain groups of would-be students. Debt aversion has the greatest impact on prospective students from low income families, the very group the government most wants to attract into higher education. [...] However, student debt has increased rapidly as a direct result of the 1998 Teaching and Higher Education Act, and is set to rise yet further following the introduction of variable tuition fees in 2006. So overall, the actual student funding system may act as a disincentive and obstacle to access and participation, especially for those from low income families who are most reliant on student loans and leave university with the highest debts. [...] The 1998 reforms of student funding, therefore, have led to a rise in the financial burden of higher education particularly for the poorest. With that rise, these students encounter increases in the financial and personal risks associated with going to university. The most disadvantaged students, and the very focus of widening participation policies, experience the greatest risks, hardship and financial pressures, all of which affect their chances of success and their ability to participate fully in university life” (p.126).

Although they do not specify the origin of the observed phenomena, the
empirical work by Brodaty et al. (2009) also contributes to stress a difference in behaviour between individuals from very educated classes and the others. The authors point out that students coming from the most educated classes (the “sons of professionals”) would paradoxically be more risk averse: they would have more to lose than the less educated classes. However, these differences are not sufficient to offset other mechanisms\(^{10}\) that induce higher private returns from educational investment for children coming from the most educated social classes (and lower working class participation in higher education). Higher fees appear then relatively harmful to university participation of the working class\(^{11}\).

These results concerning the relation between social classes and propensity for university enrolment are close to those obtained in sociology by Bourdieu (1974). Except for the most privileged classes (that feel secure enough to dare difficult educational paths), they underline that the middle class most often restrict its educational investment in order to secure returns expected from studies rather than taking real educational risks\(^{12}\). At the same time, disadvantaged

\(^{10}\) According to the authors, these mechanisms includes “unobservable inter-generational transfers” (that are partly explained in the other articles cited above).

\(^{11}\) “Simulations also show that the impact of higher education costs (i.e. tuition fees and other costs) on higher education enrolment is important, affecting more the students whose parents are less educated” (Brodaty et al. (2009), p.28).

\(^{12}\) Bourdieu (1974), p.14. Bourdieu also notes that the educational system is built “at every crossroads of the curriculum” in order to distinguish the “stockholder approach” of the middle class (the “petits bourgeois” class in Bourdieu’s vocabulary) and the “speculation in order to maximise profit” approach of the privileged classes: “The most risky courses of studies, that are often the most prestigious ones, have always a kind of less glorious alternative, abandoned for those who do not have enough (economic, cultural or social) capital to risk losing everything
social classes are widely (self-)excluded from higher education (and particularly the most prestigious).

Therefore it seems reasonable to suggest that students from disadvantaged backgrounds are more reluctant to run into debts than students from privileged backgrounds who often do not need to borrow and who enjoy domestic security. As a consequence, the former will tend to get into debt only in the case of a very high self-assessment of their talent. Conversely, the latter will use debt rather easily, if they need.

Thus, it seems wrong to consider that giving access to credit in order to finance studies enables effective access to these courses: “The rights given by law are only the explicit, guaranteed and legitimate form of this set of appropriate opportunities, of monopolised possibles, by which the current power struggles launch themselves into the future, controlling for their part the current talents. [Thus the dispossessed classes] tend to proportion their educational investment in functions of the promised profits, thus anticipating the risks of the system”\textsuperscript{13}.

The gap existing between those two dimensions is actually rather close to that of Sen (1992)’s analysis in terms of "capability".

The rationality assumption by Gary-Bobo and Trannoy (2008) is therefore particularly inappropriate to the analysis of the educational investment field since rationality is in fact “the output of a particular economic condition, the one that defines the possession of economic and cultural capital which is necessary 

to catch efficiently the 'potential opportunities' that are formally given to all, but that are really available only those having the necessary instruments to grab them"\textsuperscript{14}. In this context and despite the low borrowing constraint, the relatively strong impact of rising tuition fees on the decisions of continuing studies (and on career choices) also tends to support the assumption that privileged and disadvantaged populations do not have the same perception of their "ability" (to succeed), i.e. of their talents.

1.3 Proposal for a new assumption

The preceding analysis allows us to show that both assumptions made by Gary-Bobo and Trannoy (2008) (the existence of imperfect information and the absence of borrowing constraints) can not be raised independently from the economic, social and cultural context in which each individual evolves. In one case as in the other, a bias appears. It is almost always in favour of socially or culturally privileged classes and unfavourable to the disadvantaged ones. This bias may be (at least partially) modelled. Keeping the hypothesis that talents are randomly distributed among the population, we introduce a bias in the perception individuals have of their talent. Instead of modelling this perception by a common and zero mean random variable, we distinguish two groups within the population:

- a group of individuals belonging to "disadvantaged" class. In this class, each individual perceives his talent with a negative bias, indicating indir-\textsuperscript{14}Bourdieu (1974), p.11.
ectly a mistaken belief in the expected returns from educational investment, a possible greater risk aversion or debt burden aversion\textsuperscript{15};

• a group of individuals belonging to "privileged" class. In this class, each individual perceives his talent with a positive or zero bias. For simplicity reason, we choose the absence of perception bias for individuals in this class.

In the next section, we analyse the impact of this change of assumption on the results outlined by Gary-Bobo and Trannoy (2008).

2 The proposed model

2.1 The population

2.1.1 Homogeneous population model

Before introducing our model, we recall Gary-Bobo and Trannoy (2008)'s assumptions and notations. The modifications we propose are presented in Section 2.1.2.

According to Gary-Bobo and Trannoy (2008)'s model, workers are divided in two categories: skilled who are graduated, and unskilled, who did not study. Unskilled workers' wage is a constant rate $w_0$. Students pay tuition charges $p$ during the first period (and do not receive wages). They become skilled workers

\textsuperscript{15}Note that debt aversion to debt depends on both risk aversion and on expected returns to educational investments. Note also that, in an way, our model catches indirectly the difficulties that the most disadvantaged population is facing because of heir specific borrowing constraints.
after completing their studies.

Skilled workers’ wage depends on a common skilled premium $K(q)$ (earned by means of education), where $q$ is the number of graduates$^{16}$. It also depends on individual’s talent (or “ability”), modelled by a random variable $\tilde{\theta}$. Taking into account a constant preference for present $r$, we can write individual infinite horizon inter-temporal utilities as follows:

- for an unskilled worker: $u_0 = w_0 + \frac{\ln(w_0)}{r}$
- for a skilled worker: $u_1 = -p + \frac{\ln(w_0) + \tilde{\theta} + K(q)}{r}$

Let $\theta = \frac{\tilde{\theta}}{\sigma_{\theta}}$ and $\Delta(q) = \frac{K(q)}{\sigma_{\theta}}$. The difference between utilities is then:

$$u_1 - u_0 = \Delta(q) + \theta - p - w_0$$

where $\theta$ is supposed to be Gaussian with zero mean and variance $\sigma_{\theta}^2$. Information on $\theta$ is supposed incomplete and asymmetric. A potential student observes a noisy signal of his ability:

$$s = \theta + \varepsilon, \text{ with } \varepsilon \sim \mathcal{N}(0, \sigma_{\epsilon})$$

whereas the university observes the tests results that provide another estimation of the ability:

$$z = \theta + \nu, \text{ with } \nu \sim \mathcal{N}(0, \sigma_{\nu})$$

$^{16}$As underlined by Gary-Bobo and Trannoy (2008), the function $K$ can by either increasing or decreasing; skilled workers can be seen in concurrence on the labour market and therefore their amount would have a negative impact on their wage; but a large amount of skilled workers could also increase high-wage job opportunities by developing a knowledge economy.
The university sets an admission standard $z_0$. If $z < z_0$, the individual is 
not allowed to apply. Otherwise, potential students choose to register or not, 
and dispose of two information sources to estimate their $\theta$: their private signal 
s and the fact that $z \geq z_0$.

Potential students apply for higher education if their expected utility as 
skilled worker ($u_1$) is higher than the utility of unskilled workers ($u_0$):

$$\mathbb{E}[u_1|s, z \geq z_0] = \mathbb{E}[\theta|s, z \geq z_0] - p + \frac{\ln(w_0)}{r} + \Delta(q) \geq u_0$$

$$\iff \mathbb{E}[\theta|s, z \geq z_0] \geq p + w_0 - \Delta(q)$$

Let $\hat{\theta} = \mathbb{E}[\theta|s, z \geq z_0]$: for a potential student, it is the expectation of his 
own ability.

Let $\theta_0 = p + w_0 - \Delta(q)$: it is the minimum expected ability below which 
applying for higher education is not worthwhile.

2.1.2 Heterogeneous population model

In this section, we present some modifications to Gary-Bobo and Trannoy 
(2008)’s model, already discussed in Section 1.3. As for an homogeneous popu-
lation, we assume that abilities are purely random: $\theta \sim \mathcal{N}(0, \sigma_\theta)$. Nevertheless, 
we introduce heterogeneity in the population: the signal is now social group- 
dependent.

In group A of individuals, with population $N_A$, coming from “privileged”class, 
the signal is the same than the one in Gary-Bobo and Trannoy (2008)’s model:

$$s_A = \theta + \varepsilon, \text{ with } \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon).$$
In group B of individuals, with population $N_B$, coming from “disadvantaged” class, a negative bias impacts the signal:

$$s_B = \theta - \delta + \varepsilon, \text{ with } \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon) \text{ and } \delta > 0.$$ 

For both groups, the university signal remains:

$$z = \theta + \nu, \text{ with } \nu \sim \mathcal{N}(0, \sigma_\nu).$$

Let $\hat{\theta}_A$ and $\hat{\theta}_B$ be the value of $\hat{\theta}$ for an individual from groups $A$ and $B$, respectively. The opportunity of higher education depends on the group and is given by:

$$\hat{\theta}_A = \mathbb{E}[\theta|s_A, z \geq z_0] \geq \theta_0$$
$$\hat{\theta}_B = \mathbb{E}[\theta|s_B, z \geq z_0] \geq \theta_0$$

Let us underline that the bias of group $B$ impacts the estimation of $\theta$. As this bias is not conscious the estimator of $\theta$ for group $B$ is the same as for group $A$, and is based on the mispecified model $s = \theta + \varepsilon$ and the correct model $z = \theta + \nu$. The estimator $\hat{\theta}$ can be decomposed as a function of $s$ and $\mathbb{1}_{z \geq z_0}$, for any potential student, whatever the group, as follows:

$$\hat{\theta} = us + v\mathbb{1}_{z \geq z_0}$$

Consider two potential students with same abilities $\theta$ and same noise $\varepsilon$ but from different groups. The individual from group $B$ estimates less beneficial to apply for higher education since:
\[ \hat{\theta}_A - \hat{\theta}_B = u s_A + v I_{z \geq z_0} - u s_B - v I_{z \geq z_0} = u \delta > 0 \]

### 2.2 “Philanthropic” university: optimal tuition fees in presence of bilateral asymmetric information

Adapting Gary-Bobo and Trannoy (2008)’s notations, we set
\[ P_A(\theta_0, z_0) = \mathbb{P}(\hat{\theta}_A \geq \theta_0, z \geq z_0) \]
the probability that a potential student from group A applies for higher education and by \( \nu_A(\theta_0, z_0) = \mathbb{E}[\theta | \hat{\theta}_A \geq \theta_0, z \geq z_0] \) the conditional expectation of \( \theta \) for the resulting skilled worker. The corresponding \( \nu_B \) and \( P_B \) for group B are defined in a similar way.

For a “philanthropic” university, the expected social surplus is given by the sum of individual expected utilities on the population (\( q = N_A P_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0) \) skilled and \( N - q \) unskilled workers) minus the cost of higher education:

\[ W = q [\Delta(q) - w_0] + N_A P_A(\theta_0, z_0) \nu_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0) \nu_B(\theta_0, z_0) - C(q) + Nu_0 \]

This social surplus is maximised with respect to the expected amount of students \( q \), the admission standard \( z_0 \) and the tuition fees \( p \) (or equivalently \( \theta_0 \)) and under the constraint that \( q = N_A P_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0) \).

Let us start by maximising \( W \) with respect to \( z_0 \) and \( \theta_0 \) for a fixed value of \( q \). The Lagrangian corresponding to this maximisation is given by:
\[ L = q [\Delta(q) - w_0] + q\nu_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0) (\nu_B(\theta_0, z_0) - \nu_A(\theta_0, z_0)) \]
\[- C(q) + Nu_0 + \lambda[q - (N_A P_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0))] \]

To simplify notation, we denote by \( \partial \) the partial derivative with respect to \( \theta_0 \). The first order condition corresponding to \( \theta_0 \) writes:
\[ \partial L = q \partial \nu_A + N_B \partial P_B(\nu_B - \nu_A) + N_B P_B(\partial \nu_B - \partial \nu_A) - \lambda(N_A \partial P_A + N_B \partial P_B) = 0. \]

Since \( q = N_A P_A + N_B P_B \),
\[ 0 = (N_A P_A + N_B P_B) \partial \nu_A + N_B P_B(\partial \nu_B - \partial \nu_A) - \lambda(N_A \partial P_A + N_B \partial P_B) \]
\[ = N_A \partial \nu_A + N_B \partial \nu_B + N_B \partial P_B \nu_B + N_A \partial P_A \nu_A - (\nu_A + \lambda)(N_A \partial P_A + N_B \partial P_B) \]

\[ \lambda = -\nu_A + \frac{\partial (N_A P_A \nu_A + N_B P_B \nu_B)}{\partial (N_A P_A + N_B P_B)} \]  

(1)

Accordingly, differentiating with respect to \( z_0 \), we obtain:
\[ \lambda = -\nu_A + \frac{\partial \nu_A + N_B \partial P_B(\nu_B - \nu_A)}{\partial \nu_A + \lambda(N_A \partial P_A + N_B \partial P_B)} \]

To simplify notations again, we define the weighted expected ability \( q\nu \) and weighted probability of applying \( NP \) as:
\[ q\nu = N_A P_A(\theta_0, z_0) \nu_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0) \nu_B(\theta_0, z_0) \]
and \( NP = N_A P_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0) \)

then the first order condition can be written:
\[
\begin{align*}
\lambda &= \frac{\partial q\nu}{\partial \nu / \partial \nu_A} - \nu_A \\
\lambda &= \frac{\partial q\nu}{\partial \nu / \partial \nu_A} - \nu_A
\end{align*}
\]
and we get the tautological equality of the ratio of partial derivatives

\[
\frac{\partial NP}{\partial \theta_0} \bigg/ \frac{\partial \theta_0}{\partial z_0} = \frac{\partial q^\nu}{\partial \theta_0} \bigg/ \frac{\partial \theta_0}{\partial z_0}
\]

The maximisation with respect to \( q \) gives:

\[
\frac{\partial L}{\partial q} = q\Delta'(q) + \Delta(q) - w_0 - C'(q) + \nu_A + \lambda = 0
\]

Since \( p = \theta_0 + \Delta(q) - w_0 \) and using Equation (1) to replace \( \lambda \), we find:

\[
0 = q\Delta'(q) + p - \theta_0 - C'(q) + \nu_A - \nu_A + \frac{\partial(N_A P_A \nu_A + N_B P_B \nu_B)}{\partial(N_A P_A + N_B P_B)}
\]

or \( p = C'(q) - q\Delta'(q) - \frac{\partial(N_A P_A \nu_A + N_B P_B \nu_B)}{\partial(N_A P_A + N_B P_B)} + \theta_0 \)

This yields to (see Appendix B):

\[
\frac{\partial(N_A P_A \nu_A + N_B P_B \nu_B)}{\partial(N_A P_A + N_B P_B)} = \theta_0 + \frac{u \delta N_B \partial B}{N_A \partial A + N_B \partial B}
\]

and therefore, at the optimum:

\[
p = C'(q) - q\Delta'(q) - \frac{u \delta N_B \partial B}{N_A \partial A + N_B \partial B}
\]

Recall that the optimal tuition fees found in the Gary-Bobo and Trannoy (2008)’s model were \( p = C'(q) - q\Delta'(q) \). They are therefore reduced by a quantity (the fraction) that comes from the need to counterbalance the bias of potential students from group \( B \). This leads to the following proposition:

**Proposition 1.** In a “philanthropic” view, in presence of heterogeneity in the population leading to underestimation of the ability for some individuals, optimal tuition fees are lower than those obtained by Gary-Bobo and Trannoy (2008)
for an homogeneous population, given the amount of student:

\[ p^* = \frac{C'(q^*) - q^* \Delta'(q^*)}{p^*_GT} = \frac{u \delta N_B \partial P_B}{N_A \partial P_A + N_B \partial P_B}. \]

**Remark.** The effect of heterogeneity increases with the relative weight of group \( B \) in the population. It converges to 0 as the group becomes negligible.

The effect of heterogeneity increases with \( u \): i.e. when potential students, deciding whether applying or not for higher education, use more their personal signal \( (s) \) than the information given by their success at the university examinations \( (z \geq z_0) \). Therefore, the information given by the test is the only way to counterbalance the psychological bias of group \( B \). The bias is sort of “balanced” by the success at the test.

\[^{17}\text{We have } \hat{\theta} = us + v1l_{z \geq z_0}.\]

\[ u = \sigma^2 V(1l_{z \geq z_0}) - \text{Cov}(s, 1l_{z \geq z_0}) \text{ and } v = \sigma^2 V(s) V(1l_{z \geq z_0}) - \text{Cov}(s, 1l_{z \geq z_0}), \]

where \( V(s) = \sigma^2 + \sigma^2_{\varepsilon} \) is the variance of \( s \),

\[ V(1l_{z \geq z_0}) = P(z \geq z_0)(1 - P(z \geq z_0)) = \left(1 - \Phi \left( \frac{z_0}{\sigma^2 + \sigma^2_{\varepsilon}} \right) \right) \left( \Phi \left( \frac{z_0}{\sigma^2 + \sigma^2_{\varepsilon}} \right) \right) \]

is the variance of \( 1l_{z \geq z_0} \) and there covariance is

\[ \text{Cov}(s, 1l_{z \geq z_0}) = \sigma^2 \int \int x1l_{x \geq x_\theta, y \geq y_\theta} \varphi \left( \frac{x}{\sigma^2} \right) \varphi \left( \frac{y}{\sigma^2_{\varepsilon}} \right) dxdy \]

where \( \varphi \) is the probability density function of the standard Gaussian distribution and \( \Phi \) its cumulative density function.

\[^{18}\text{Nevertheless, the problem may only be postponed: for the same reasons that led to introduce } \delta, \text{ numerous papers (notably in sociology) reveal the presence of a bias, adverse to students from “disadvantaged” groups. This bias is said to be due to “social codes” needed to pass examinations (and } z \geq z_0 \text{ would therefore be biased as well). This would advocate}\]

\[ 22 \]
2.3 Rent maximising university: optimal tuition fees in presence of bilateral asymmetric information

We consider here the case of a non "philanthropic" university: it is not anymore the social utility that is maximised, but the university profits. Of course, the indirect social benefit obtained is expected to be poor with respect to the one in previous section\(^1\).

We maximise then the profit function of the university under the constraint

\[ q = N_A P_A(\theta_0, z_0) + N_B P_B(\theta_0, z_0). \]

The Lagrangian writes as follows:

\[ L = qp - C(q) + \lambda (q - N_A P_A - N_B P_B) = q[\theta_0 + \Delta(q) - w_0] - C(q) + \lambda (q - N_A P_A - N_B P_B). \]

Computing the partial derivative with respect to \(q\), we find:

\[ 0 = \theta_0 + \Delta(q) + q\Delta'(q) - w_0 - C'(q) + \lambda \]

\[ \Leftrightarrow \quad p = C'(q) - q\Delta'(q) - \lambda. \]

The derivative with respect to \(\theta_0\), gives:

\[ 0 = q - \lambda (N_A \partial P_A + N_B \partial P_B) \Leftrightarrow \quad \lambda = \frac{q}{N_A \partial P_A + N_B \partial P_B}. \]

Consider the situation at the optimum. If the university could attract more students by rising the tuition fees, its utility could be augmented. Therefore, at the optimum, the expected number of students is a decreasing function of \(\theta_0\) for a large test, eventually compulsory, as the French “baccalauréat”. Further research should investigate this issue.

\(^1\)In their paper, Gary-Bobo and Trannoy (2008) do not model competition between universities. Therefore, the case of a non “philanthropic” university leads necessarily to a sub-optimal situation of monopolistic prices.
and then \(N_A \partial P_A + N_B \partial P_B < 0\). As \(q > 0\), we get \(\lambda < 0\). This leads to the following proposition:

**Proposition 2.** From a social point of view, tuition fees are sub-optimal, for both groups: they are too high for group A (this result also stands for an homogeneous population) and a fortiori for group B.

\[
p^* > C'(q^*) - q^* \Delta'(q^*)
\]

In addition, the optimal admission standard \(z_0^*\) must be set such that it does not impact the amount of students\(^{20}\).

### 2.4 One-sided asymmetric information

We now follow Gary-Bobo and Trannoy (2008) by studying the case in which the signal of a “philanthropic” university (\(z\)) is supposed to be public.

In such a case, since \(z\) is known, potential students can use more information than previously to estimate if higher education is beneficial. Since in this model, social utility and individual utility are varying in the exactly same way through the evolution of wage, individual choices are also socially optimal.

The resulting proposition is similar to the one of section 2.2.

**Proposition 3.** In the case of a “philanthropic” university, for a heterogeneous

\(^{20}\)Indeed, computing the derivative of \(L\) with respect to \(z_0\), we find that at the optimum

\[
N_A \frac{\partial P_A}{\partial z_0} + N_B \frac{\partial P_B}{\partial z_0} = 0.
\]
population, if \( z \) is public then optimal tuition fees are less than those \((p^*_{\text{GT}})\) obtained in the case of an homogeneous population, for a fixed amount of students:

\[
p^* = C'(q^*) - q^* \Delta'(q^*) - \frac{\alpha \delta N_B \partial P_B}{N_A \partial P_A + N_B \partial P_B},
\]

where \( \alpha \) measures the information on \( \theta \) coming from the personal signal \( s \).

Since \( z \) is directly available to potential students, and not anymore indirectly through \( \mathbb{1}_{z \geq z_0} \), the effect discussed in the previous section is reinforced: as information coming from \( z \) increases (i.e. \( \sigma_\nu \) decreases with respect to \( \sigma_\varepsilon \)), potential students from group B become aware of their talent. In the limit case where the test brings complete information on the talent, the effect of the psychological bias vanishes (\( \alpha = 0 \)). Optimal tuition fees are then equivalent to the case of an homogeneous population.

\( ^{21} \)The coefficient \( \alpha \) is given by the estimation of \( \theta \) from \( s \) and \( z \) (instead of \( s \) and \( \mathbb{1}_{z \geq z_0} \)):

\[
\bar{\theta} = \mathbb{E}[\theta | s, z] = \alpha s + \beta z.
\]

with

\[
\alpha = \frac{\sigma^2_\theta V(z) - \text{Cov}(s, z)}{V(s)V(z) - \text{Cov}^2(s, z)} = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon + \sigma^2_\nu}.
\]

Note nevertheless that a social rank bias can appear on the test itself (either on the inscription to the test or on the possession of the “social codes” needed to succeed the test). In such a case, even a ideal test revealing perfectly the talent of individuals of group \( A \) would not be sufficient to select efficiently.
3 Discussion and perspectives

The literature highlights distorted behaviours of students, depending on the social class they belong to: While individuals from some social groups tend to estimate more precisely (or even over-estimate) their ability and the expected returns of educational investment, other social groups under-estimate those dimensions. Based on this observation, we proposed in this article a more realistic assumption than the one adopted by Gary-Bobo and Trannoy (2008) concerning the population characteristics.

Following our hypothesis, it appears that tuition fees are inefficient as a tool of selection for at least two reasons: (i) they tend to move away the good students that would have "deserved" to go to university but that have underestimated their talents; (ii) they also tend to include low talented students coming from privileged families.

Relying on this bias, highlighted by a rich literature, we have shown that tuition fees should be set below the level that would be fixed for a homogeneous population and the same volume of students. This conclusion is particularly important since empirical studies have deduced that most of the population does not really face borrowing constraints when making a decision for further studies. This decision depends on other criteria, primarily social ones, that are strong enough disincentives for university enrolment and for running into debt. They can distort the corresponding educational choices and job opportunities. Low enough tuition fees appear then as a key and necessary (even though not sufficient) condition in order to achieve a socially optimal equilibrium.
However, further research has to be done in order to guarantee that assumptions are strong enough to implement recommendations coming from such a model. Our work should also be extended in order to determine what should concretely be tuition fees: Are they higher than those applied today? Lower? Even negative? Are they still relevant to select potential students applying at university? Those dimensions depend in particular on the parameters including the population distribution among the two social groups as well as on the characteristics of these populations.

Another perspective concerns the likely endogenous nature of individuals’ abilities, modelled by $\theta$: If individuals’ talents are not only "revealed" through education but also "developed" by it, in a dynamic perspective, the fees should be even lower in order to keep in the system a population initially low "gifted" but improving their potential by studying.

However, staying with exogenously distributed and static abilities as in Gary-Bobo and Trannoy (2008)’s model, the social welfare would appear particularly important since the educational system integrates stronger incentives for "very good but very poor" students and disincentives for “low talented but very rich" students.

Such a remark suggests to develop further research on the selection relying on entry examinations. It also suggests research perspectives concerning possibly more efficient (not linear) pricing mechanisms. These mechanisms can be considered directly (by adjusting fees according to various criteria) or indirectly (through grants). Some institutions are already testing this approach, fees be-
ing based on social criteria (parental income) or "merit" ones since attracting the best students creates positive externalities for universities (reputation effect) and students (peer effect). The level of tuition fees could also be designed according to the job and to the real income of the student after graduation.

Finally, even if Gary-Bobo and Trannoy (2008) consider tuition fees only as a selection tool, further work could be done exploring the other (also questionable) justifications of fees provided by the literature. These are of three types: Incentive, contributory and redistributive ones.

(i) Incentive justifications. A first incentive has been reviewed in this article: It consists in using tuition fees to exclude the worst students from higher education and to encourage the best ones to get enrolled. Another incentive is linked to the motivation of students and teachers. Indeed, it would become very expensive for students not to work (or to work little) while paying high fees for studies. At the same time, teachers would be involved because of students’ motivation and demands, and possibly by a system of bonuses (partly funded by the fees). (ii) Contributory justifications. By paying fees, students contribute to finance universities and thus to increase the quality of their education (by recruiting the best teachers, by funding research, by improving working conditions). (iii) Redistributive justifications. The fees could play a redistributive role since universities are mostly frequented by the upper social classes.

Each of these dimensions require a specific discussion and further research in order to consider whether, in other forms or for other reasons, the introduction of “significant” tuition fees remains relevant. However, as far as shown in this
article, Gary-Bobo and Trannoy (2008)’s views, results and recommendations (see also Gary-Bobo and Trannoy (2005b)) about fees as a “good” tool of selection have to be put into perspective. Further work has to be done before providing definitive conclusions. It has also be done before implementing “high tuition fees policies”, especially in countries of still “free” education like in France, since this implementation will undoubtedly lead to an in-depth transformation of their economic, social and cultural patterns of development.

A Expected global talent

In the section, we investigate the expected mean talent in presence of heterogeneity. The aim is to compare it with the situation of an homogeneous population and therefore we will assume that heterogeneity is not too important, in order to deal with marginal modifications. For the same reason, we assume that $z_0$ is fixed to its optimal value for the homogeneous population.

Let $\theta_0^{GT}$ and $\theta_0^*$ be the optimal values obtained for an homogeneous population and an heterogeneous population, respectively. Let $q^{GT}$ and $q^*$ be the corresponding optimal amounts of students, supposed to be close to each other.

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22 This article is based on an earlier version of the Gary-Bobo and Trannoy (2008)’s work, published as a working paper (Gary-Bobo and Trannoy (2005a)).
\[ q^* \nu_A(\theta_0^*, z_0) + N_B P_B(\nu_B(\theta_0^*, z_0) - \nu_A(\theta_0^*, z_0)) - q^{GT} \nu(\theta_0^{GT}, z_0) \]
\[ = q^* \mathbb{E} \left[ \theta \left| \hat{\theta}_A \geq \theta_0^*, z \geq z_0 \right. \right] - q^{GT} \mathbb{E} \left[ \theta \left| \hat{\theta}_A \geq \theta_{0^*}^{GT}, z \geq z_0 \right. \right] \]
\[ + N_B P_B \left( \mathbb{E} \left[ \theta \left| \hat{\theta}_A - u \delta \geq \theta_0^*, z \geq z_0 \right. \right] - \mathbb{E} \left[ \theta \left| \hat{\theta}_A \geq \theta_0^*, z \geq z_0 \right. \right] \right) \]
\[ = q^* \nu_A(\theta_0^*, z_0) - q^{GT} \nu_A(\theta_{0^*}^{GT}, z_0) + N_B P_B \left( \nu_A(\theta_0^* + u \delta, z_0) - \nu_A(\theta_{0^*}^{GT}, z_0) \right) \]

(2)

Notice that

\[ \theta_{0^*}^{GT} - \theta_0^* = p^{GT} - p^* + \Delta(q^{GT}) - \Delta(q^*) \]
\[ = C'(q^{GT}) - C'(q^*) - (q^{GT} - q^*) \Delta(q^{GT}) + \frac{u \delta N_B P_B}{N_A \partial P_A + N_B \partial P_B} + (q^{GT} - q^*) \Delta'(q^{GT}) \]
\[ \approx \frac{u \delta N_B P_B}{N_A \partial P_A + N_B \partial P_B} \]

assuming that the marginal cost of an additional student is fixed.

The first term of the Equation 3 writes:

\[ q^* \nu_A(\theta_0^*, z_0) - q^{GT} \nu_A(\theta_{0^*}^{GT}, z_0) = q^* \left( \theta_{0^*} - \theta_{0^*}^{GT} \right) \nu(\theta_{0^*}^{GT}, z_0) + (q^{GT} - q^*) \nu(\theta_{0^*}^{GT}, z_0) \]
\[ = -q^* \frac{u \delta N_B P_B}{N_A \partial P_A + N_B \partial P_B} \nu(\theta_{0^*}^{GT}, z_0) + (q^{GT} - q^*) \nu(\theta_{0^*}^{GT}, z_0). \]

Consider now the second term:

\[ N_B P_B \left( \nu_A(\theta_0^* + u \delta, z_0) - \nu_A(\theta_{0^*}^{GT}, z_0) \right) = N_B P_B \left( \theta_{0^*} + u \delta - \theta_{0^*}^{GT} \right) \partial \nu(\theta_{0^*}^{GT}, z_0) \]
\[ = N_B P_B u \delta \frac{N_A \partial P_A - N_B \partial P_B}{N_A \partial P_A + N_B \partial P_B} \partial \nu(\theta_{0^*}^{GT}, z_0). \]

The evolution of the global talent is then

\[ q \nu_A(\theta_0^*, z_0) + N_B P_B \left( \nu_B(\theta_0^*, z_0) - \nu_A(\theta_0^*, z_0) \right) - q^{GT} \nu(\theta_{0^*}^{GT}, z_0) \]
\[ = q \left( u \delta N_B P_B \nu(\theta_{0^*}^{GT}, z_0) + N_B P_B u \delta \left( N_A \partial P_A - N_B \partial P_B \right) \partial \nu(\theta_{0^*}^{GT}, z_0) \right) \frac{N_A \partial P_A + N_B \partial P_B}{N_A \partial P_A + N_B \partial P_B} \]
\[ + (q^{GT} - q^*) \nu(\theta_{0^*}^{GT}, z_0) \]

30
B The effect of heterogeneity on the optimal tuition fees

Let us simplify the term introduced by the heterogeneity of the population:

\[
\frac{\partial(N_A P_A \nu_A + N_B P_B \nu_B)}{\partial(N_A P_A + N_B P_B)}
\]

The denominator is simply \(N_A \partial P_A + N_B \partial P_B\). Let us consider the numerator.

Notice that

\[
P_A \nu_A = \mathbb{P}\left(\hat{\theta}_A \geq \theta_0, z \geq z_0\right) \mathbb{E}\left[\theta \mid \hat{\theta}_A \geq \theta_0, z \geq z_0\right] = \int \theta \mathbb{1}_{\theta \geq \theta_0, z \geq z_0} f_{\hat{\theta}_A}(\theta) d\theta
\]

Differentiating with respect to \(\theta_0\):

\[
\partial P_A \nu_A = - \int \theta \mathbb{1}_{\theta = \theta_0, z \geq z_0} f_{\hat{\theta}_A}(\theta) d\theta
\]

\[
= - \mathbb{E}\left[\theta \mid \hat{\theta}_A = \theta_0, z \geq z_0\right] \mathbb{P}\left(\hat{\theta}_A = \theta_0, z \geq z_0\right)
\]

\[
= \mathbb{E}\left[\theta \mid \hat{\theta}_A = \theta_0, z \geq z_0\right] \partial P_A
\]

The same calculation leads to

\[
\partial P_B \nu_B = \mathbb{E}\left[\theta \mid \hat{\theta}_B = \theta_0, z \geq z_0\right] \partial P_B
\]

Since \(\hat{\theta}_A\) is unbiased,

\[
\mathbb{E}\left[\theta \mid \hat{\theta}_A = \theta_0, z \geq z_0\right] = \theta_0 \text{ and } \partial P_A \nu_A = \theta_0 \partial P_A.
\]

On the contrary, \(\hat{\theta}_B\) has a non null bias equals to \(-u\delta\) and then

\[
\mathbb{E}\left[\theta \mid \hat{\theta}_B = \theta_0, z \geq z_0\right] = \theta_0 + u\delta \text{ and } \partial P_B \nu_B = (\theta_0 + u\delta) \partial P_B.
\]

Finally, we get:

\[
\frac{\partial(N_A P_A \nu_A + N_B P_B \nu_B)}{N_A \partial P_A + N_B \partial P_B} = \frac{N_A \delta \partial P_A + N_B (\theta_0 + u\delta) \partial P_B}{N_A \partial P_A + N_B \partial P_B} = \theta_0 + \frac{u\delta N_B \partial P_B}{N_A \partial P_A + N_B \partial P_B}
\]
References


