Modelling sovereign default risk: Evidence from Argentina

Sy-Hoa HO*

Abstract
The objectives of this paper are to review the Merton’s credit risk model and the methodology to calculate the sovereign asset value and sovereign asset volatility of emerging countries by using the sovereign default risk model. We examine the sovereign default model of Gray et al. (2007) for the empirical case of Argentina on the period from 1997 to 2009, we obtain results of Argentina’s default probability that tend fluctuate to the real Argentinian economy.

Key words: sovereign default risk, probability of default
JEL classification: G32, F34

Résumé
Les objectifs de ce papier sont d'examiner le modèle de risque de crédit de Merton et la méthodologie afin de calculer la valeur des actifs et la volatilité souverain des pays émergents en utilisant le modèle du risque de défaut souverain. Ainsi nous utilisons le modèle de Gray et al. (2007) pour le cas de l'Argentine sur la période 1997-2009, nous trouvons une tendance de probabilité de défaut de l'Argentine qui consiste à l'économie réel.

Mots clés : risque de défaut souverain, probabilité de défaut.
JEL classification: G32, F34

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Abstract

The objectives of this paper are to review the Merton’s credit risk model and the methodology to calculate the sovereign asset value and sovereign asset volatility of emerging countries by using the sovereign default risk model. We examine the sovereign default model of Gray et al. (2007) for the empirical case of Argentina on the 1997-2009 periods, we obtain results of Argentina’s default probability that tend fluctuate to the real Argentinian economy.
1 Introduction

The volatility of the world economy is more complex and more difficult to forecast. The collapse and chained bankruptcy of the financial system caused the sovereign default of many countries that depends on strong currencies, like the U.S. Dollar. In fact, the financial crisis in Thailand (1997) derived from the default of large financial institutions and loss of its liquidity, together with the withdrawal of foreign investment in the country. In addition, globalization is one of the causes that lead to contagion crisis in Southeast Asian countries, such as Indonesia and Malaysia, followed by Russia and Latin American countries, for example, Brazil (2002) and Argentina (2001). Consequently, public debt management has become the first priority to stabilize the economy.

Credit risk models have been used in the lending activities of commercial banks. The original structural models began with Merton’s credit risk model (Merton, 1974) based on the option pricing theory of Black and Scholes (1973). The model allows calculating the value of a firm’s asset and asset volatility by using information of firm’s liabilities on the firm’s balance sheet. This model is also called Contingent Claims Analysis (CCA) model, and this was improved by Black and Scholes (1973), Longstaff & Schwartz (1995), Kealhofer-Merton-Vasicek (KMV, 2000) and commercialized by Moody’s. This idea was transposed from firm level to country level to calculate the sovereign asset value and sovereign asset volatility. The sovereign default risk model was developed by Gray and Malone (2008) who use macroeconomic and the option pricing theory. More precisely, they applied sovereign CCA balance sheet which interlinks balance sheet of government and authorities. The sovereign default risk model has successfully been applied to some emerging countries, such as Brazil (Gray et al., 2007; Gray and Malone, 2008) and Turkey (Souto et al., 2007).

This paper is organized as follows. In the second section, we review the Black & Scholes formula. In section 3, we present the Merton credit risk model; in section 4, we explain how to transpose from credit risk to sovereign default risk, and how to apply a sovereign CCA balance sheet model for emerging countries. In the next section, we show an empirical result for the case of Argentina for the period 1997-2009. We conclude in the last section.

2 The Black & Scholes Formula

In the trend of investment boom and globalization, the development of financial instruments is considered as one of the five pillars of stable growth in the U.S. economy. The new financial instruments on the financial market are derivative financial instruments that not only allow banks to prevent risks but also make a profit following arbitrage.

A derivative instrument is a contract between two parties that specifies conditions (especially the dates, resulting values of the underlying variables, and notional amounts) under which payments are to be made between the parties (Hull, 2008).

One of those derivatives contracts is the option contract. This is a financial contract between two parties, the buyer and the seller, that gives the buyer the right, but not the
obligation to buy/sell an underlying asset from the seller at a pre-determined price \( K \), on or before pre-determined future time \( T \). If the right is valid on the date \( T \), but not before is called a European option, otherwise if the right is valid before the time \( T \) it is called an American option. In this paper, we will only use European options contracts. There are two types of option contract: Call option and Put option.

The Black & Scholes model is one of the fundamental concepts of modern financial theory. Developed in 1973 by Black and Scholes (1973), it is considered as a benchmark to determine the option price that has been used widely today. In 1997, Merton and Scholes received the Nobel Prize in economics. This model includes the formula of Black & Scholes which gives the price of the European option.

Merton analyzed security price dynamics and asset dynamics by using a stochastic process where the stochastic process is a random process indexed by time. He used continuous time to price the security price/asset dynamic. There are two principal stochastic processes in continuous time that we use in this paper: Brownian motion and Itô’s process. Brownian motion is a stochastic process where the value of the stock may be positive or negative. Indeed, we have a variable \( X \) that follows a Brownian motion with expected rate of return (drift) \( \mu \) and volatility \( \sigma \) given by: 

\[
dX = \mu dt + \sigma dW
\]

The second one is an Itô’s process of \( X \) variable given by: 

\[
dX = \mu(t, X)dt + \sigma(t, X)dW
\]

We have Itô’s lemma of a function \( f(t, X) \) by:

\[
df(t, X) = \left[ \frac{\partial f}{\partial t} + \frac{\partial f}{\partial X} \mu(t, X) + \frac{\partial^2 f}{\partial X^2} \sigma^2(t, X) \right] dt + \frac{\partial f}{\partial X} \sigma(t, X)dW
\]

where \( dW \) is a Brownian process.

If \( \mu(t, X) = \mu X \) and \( \sigma(t, X) = \sigma X \) then the function \( df(t, X) \) in equation (1) becomes 

\[
dX/X = \mu dt + \sigma dW
\]

this new function is named Geometric Brownian Motion (GBM). Therefore, the value of the underlying asset dynamics describes a GBM.

The Call option is one of three assets dynamics with risk-free interest rate and underlying risky asset in the continuous-time market, and the Call option follows an Ito’s lemma process.

For an option pricing contract, Merton (1990) used several assumptions:

- There is a perfect market
  - There is no transaction cost, tax.
  - All participants in the market can lend and borrow at the same risk-free rate; This risk-free rate is exogenous and constant during the life cycle of the option.
  - There exists a sufficient number of investors; each investor believes that he can buy and sell assets at the desired amount.
  - It allows short-selling of all assets

\footnote{\( f(t, X) \) is at least twice differentiable in \( X \) and one differentiable in \( t \)}

\footnote{short-selling is the practice of selling borrowed securities when securities price down, hope that the price will down so that borrower can buy it back at a lower price to return the borrower to the lender.}
• There is no arbitrage opportunities (without risk).

• With an instantaneous risk-free rate \( r \) at any period, the price of a risk-free discount bond paying one dollar at time \( T \) in the future is \( p(T) = e^{-rT} \).

• The trading of the securities is continuous.

• The value of the underlying asset follows a GBM written by: \( dA = \mu Adt + \sigma AdW \); where: \( A \) is value of the underlying asset; \( \mu \) is a drift; \( \sigma \) is volatility of the underlying asset; \( dW \) is a standard Gauss-Wiener process.

Under these assumptions, we review the Call, Put formula:

The Call option is a financial contract which gives the buyer the right, not the obligation, to purchase a security at a pre-determined price (strike price) \( K \) on the future date \( T \). In return, the buyer is required to pay the seller a fee that is called call premium \( (p) \). The buyer of the Call option hopes that the price of underlying asset will rise in the future. If the stock price rises as expected, then the buyer will exercise this contract, since he has invested only a small fee. Otherwise, if the stock price is below the price \( K \), we do not exercise the Call option contract because the pay-off is negative. The pay-off of a Call option is: \( max(0, A - K) \) where \( A \) is the value of the underlying asset at the time \( T \).

We have the equation for the Call option price:

\[
C = AN(d_1) - Ke^{-rT} * N(d_2)
\]

where

\[
d_1 = \frac{ln(A/K) + (r + 0.5\sigma^2)T}{\sigma \sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma \sqrt{T}; \quad N() \text{ is the cumulative standard normal distribution, } r \text{ is a continuous risk-free interest rate, } \sigma \text{ is volatility of the underlying asset, } T \text{ is time.}
\]

We can see in Figure 1 below that: when the stock price fluctuates from 0 to \( K \), the buyer does not exercise the Call option contract. The graph is represented by a line that is parallel with the horizontal axis from 0 to \( K \). When the price is greater than \( K \), the buyer will exercise a Call option contract. In this case, the graph is represented by the 45 degree line.

The put option is a financial contract that gives the buyer the right, not the obligation to sell a security at a pre-determined price (strike price) \( K \) on the future date \( T \). In return, the buyer is required to pay the seller a fee that is called put premium \( (p) \). The buyer of the put option hopes that the price of the underlying asset will fall in the future. If the stock price falls as expected, then the buyer will exercise this contract, the buyer will benefit as he has to invest a small fee. Otherwise, if the stock price is greater than the prices \( K \), we don’t exercise the put option contract because the pay-off is negative. The pay-off of a put option is: \( max(0, K - A) \) where \( A \) is the value of underlying asset at the time \( T \).

We have the equation for the put option price:

\[
P = -AN(d_1) + Ke^{-rT} * N(d_2)
\]
where \( d_1 = \frac{Ln\left( \frac{A}{K} \right) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} \) and \( d_2 = d_1 - \sigma\sqrt{T} \); \( N() \) is cumulative standard normal distribution, \( r \) is continuous risk-free interest rate, \( \mu \) is volatility of underlying asset, \( T \) is time.

In Figure 1 below: when the stock price fluctuates from 0 to \( K \), the buyer exercises the put option contract, the graph line is represented by the 45 degree line. When the price is greater than \( K \), then the buyer will not exercise put option contract, the graph is represented by the parallel line with the horizontal axis from 0 to \( K \).

Figure 1: Profit of buyer/seller for Call/Put option

Source: Hull (2008)
3 The credit risk model

Credit risk is a popular term which is used in the banking and finance sector. In fact, one of the main activities of commercial banks is lending activities which have always a credit risk. Credit risk is a risk in which a borrower will have no capacity to pay-off his debt at maturity. Default occurs when a debtor does not pay his debt at maturity. Or we can understand that debt default occurs when the borrower has not made a scheduled payment of interest or principal. Hence, the banks want to protect themselves against credit risk, so that they require generally that the borrower has a collateral asset (housing, stocks...) in order to access the credit market. At maturity, if the borrower does not pay-off its debt, the banks will process collateral assets for payment of debts. Thus, credit risk is an important factor we have to understand strongly. Simultaneously, banks might analyse, evaluate, manage this type of risk to avoid loss of liquidity and its default.

A powerful model to evaluate default risk is Merton’s structural model [Merton 1974]. This model was developed from assumptions of option pricing of Black and Schole (1973) described in the first section. Merton’s model is based on the option pricing theory of Black and Scholes (1973) to explain the relationship between equity value and derivative option (Put, Call). The purpose of Merton’s model is to quantify the asset value and asset volatility by using information on debt and equity, so that we can determine the probability of default of loans.

The Merton model for credit risk has two steps:

• Firstly, the Black & Scholes’ equation for an European Call option is applied to estimate the value of equity.

• Secondly, the firm’s equity is to estimate asset value and asset volatility.

Firstly: the Black & Scholes’ equation for an European Call option is applied to estimate the value of equity.

This model is based on corporate balance sheet and structural capital. Suppose at time $T$, a firm has an asset $A_T$ that is financed by equity $E_T$ and zero-coupon debt $D_T$. The capital structure represents by the following formula: $A_T = E_T + D_T$

<table>
<thead>
<tr>
<th>Asset</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_T$</td>
<td>Equity $E_T$</td>
</tr>
<tr>
<td></td>
<td>Zero-coupon debt $D_T$</td>
</tr>
</tbody>
</table>

Table 1: Firm’s balance sheet

In the Merton model, he supposed that the market value of the firm’s asset dynamics $A_T$ follows the GBM:

$$ dA = rAdt + \sigma_A dW $$

or

$$ \frac{dA}{A} = rdt + \sigma_A dW $$

or
where: $A, dA$ are respectively the firm’s asset value and the change of asset value; $\mu_A, \sigma_A$ are the expected rate of return of the firm’s asset value and its volatility; $dW$ is a Wiener process.

In the original Merton’s model, the default risk appears when the asset value ($A_T$) falls below the face value of all debt ($D_T$) at maturity ($T = 1$). A strong assumption of this model is that default only occurs at maturity. This assumption is like as a characteristic of zero-coupon bond where the investor receives one payment at maturity. The face value of all debt represents the promised payments of the debt. At maturity $T$, the firm will pay-off its debt, and there are 2 situations:

- If $A_T < D_T$, the asset of this firm is not sufficient to payoff its debt, so this firm will default at time $T$.
- If $A_T \geq D_T$, the firm will pay-off its debt to creditors at time $T$. The value of equity equals to assets minus debt, otherwise the equity value is equal to zero. The value of equity can be written as $E_T = \max(A_T - D_T, 0)$.

This formula is related to the European Call option contracts and is the first step of Merton model. Indeed, we can explain this as follows: creditors hold a bond issued by a firm, we can assume that creditors have the right, not the obligation, to receive the bond of this firm. Applying option pricing theory, we exercise a Call option if the value of the firm’s asset is greater than value of total debt (total debt is the strike price), otherwise if the value of a firm’s asset is below the value of total debt, we will not exercise a Call option. So, the equity value ($E$) can be interpreted exactly as a Call option.

The value of Call price for equity is computed as:

$$E = AN(d_1) - De^{-rT} * N(d_2)$$

Where $d_1 = \frac{\ln(A/D) + (r + 0.5\sigma^2)(T)}{\sigma\sqrt{T}}$; $d_2 = d_1 - \sigma\sqrt{T}$; $A$ is the firm’s asset value, $D$ is total debt, $r$ risk-free interest rate, $\sigma$ is the firm’s asset volatility, $T$ is maturity.

**Secondly**: the firm’s equity is to estimate the asset value and asset volatility.

We can apply the Itô’s lemma for the Call price dynamics $C(t, A)$ on the value of firm’s asset $A$:

$$dC(t, A) = [\frac{\partial C}{\partial t} + \frac{\partial C}{\partial A} \mu_A + \frac{\partial^2 C}{\partial A^2} \sigma_A^2 A]dt + \frac{\partial C}{\partial A} \sigma_A A dW$$

The dynamics for equity follows a GBM process:

$$dE = \mu_E dt + \sigma_E dW$$

According to Black & Scholes equation, the delta for a Call is given by:

$$\Delta = \frac{\partial C}{\partial A} = N(d_1)$$
where \( d_1 = \frac{\ln\left(\frac{A}{D}\right) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} \)

Because we can consider the equity as a Call option, we combine formulas (4),(5),(6) for the standard Gauss-Wiener process \( dW \) factor to obtain:

\[
\sigma E E = N(d_1)\sigma A A
\]  

(7)

The outputs of this model find the implicit firm’s asset value \( A \) and the firm’s asset volatility \( \sigma_A \) by using 2 equation (3),(7). In these two equations, we know the equity value \( E \), equity volatility \( \sigma_E \), debt value \( D \), interest rate \( r \) and maturity \( T \).

\[
\begin{cases}
E = AN(d_1) - De^{-rT} * N(d_2) \\
\sigma_E E = N(d_1)\sigma A A
\end{cases}
\]

Default Probability

From equation (2), the process of asset \( A_T \) at the time \( T \) can be calculated from the asset value at time 0, is written by:

\[
A_T = A_0 \exp[(r - 0.5\sigma^2_A)T + \sigma_A \varepsilon \sqrt{T}]
\]

(8)

where \( \varepsilon \) is the random component of a normal random variable \( N(0,1) \).

The probability of default occurs when the firm’s asset value \( A_T \) is less than debt value \( D_T \) at maturity \( T \). The debt value is the promised payments value that are the present value of the debts discounted at the risk-free rate.

The probability of default under the neutral-risk \( P \) is given by:

\[ P(A_T < D_T) \equiv P(lnA_T < lnD_T) \]

we have:

\[
ln(A_T) = ln(A_0) + (r - 0,5\sigma^2_A)T + \sigma_A \varepsilon \sqrt{T}
\]

\[
P(lnA_T < lnD_T) = Pr(ln(A_0) + (r - 0,5\sigma^2_A)T + \sigma_A \varepsilon \sqrt{T} < lnD_T)
\]

\[
= P\left(- \frac{Ln\left(\frac{A_0}{D_T}\right) + (r - 0,5\sigma^2)T}{\sigma\sqrt{T}} > \varepsilon\right)
\]

\[
= P(-d_2, \varepsilon)
\]

where \( d_2, \varepsilon = \frac{Ln\left(\frac{A_0}{D_T}\right) + (r - 0,5\sigma^2)T}{\sigma\sqrt{T}} \)

Because the random component is normally distributed, \( \varepsilon \sim N(0,1) \), comparing with the above result, we can define the probability of default equal to value of \( N(-d_2) \).

For example:

Assuming that if we know the firm’s equity value \( E=27.140 \) $; \( D=393.835 \) $; \( \sigma_E = 0.254 \) risk-free rate \( r = 5 \% \), \( T = 1 \). So we have four equations:
\[
\begin{align*}
27.140 &= AN(d_1) - 393.835e^{-0.05} * N(d_2) \\
27.140 * 0.254 &= N(d_1)\sigma_A A \\
d_1 &= \frac{Ln(\frac{A}{393.835}) + (0.05 + 0.5\sigma_A^2)}{\sigma_A} \\
d_2 &= d_1 - \sigma_A
\end{align*}
\]

We find the firm’s asset value \( A = 401.7674 \) $, \( \sigma_A = 0.2565 \) and \( d_2 = 0.573 \); The default probability of this firm: \( N(-d_2\sigma) = 44.2582 \) %.

Because of the above mentioned assumptions, the Black-Scholes-Merton model has 4 main restrictions as follows:

- The default can occur only at maturity date of debt.
- There is fixed default point that is equal to total debt.
- There is a constant risk-free rate.
- The asset volatility is constant.

### 3.1 The variation of default probability

In this section, we present the variation of default probability (DP) when distress barrier, risk-free rate, maturity vary.

We can see in Table 2, 3, 4: I set a fixed asset value equal to 100 $; the distress barrier fluctuates from a very low value (9 $) to a very high value (99 $); risk-free risk varies from 5 % to 20 %; maturity varies from 1 year to 10 year.

**Table 2: Case 1: increasing \( r \) and \( T \)**

<table>
<thead>
<tr>
<th>V</th>
<th>D</th>
<th>( r )</th>
<th>( \sigma_V )</th>
<th>T</th>
<th>( d_2 )</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>75</td>
<td>0.05</td>
<td>0.4</td>
<td>1</td>
<td>0.6442</td>
<td>25.9721 %</td>
</tr>
<tr>
<td>100</td>
<td>75</td>
<td>0.05</td>
<td>0.4</td>
<td>5</td>
<td>0.1539</td>
<td>43.8831 %</td>
</tr>
<tr>
<td>100</td>
<td>75</td>
<td>0.05</td>
<td>0.4</td>
<td>10</td>
<td>-0.0097</td>
<td>50.3884 %</td>
</tr>
<tr>
<td>100</td>
<td>75</td>
<td>0.1</td>
<td>0.4</td>
<td>1</td>
<td>0.7692</td>
<td>22.0885 %</td>
</tr>
<tr>
<td>100</td>
<td>75</td>
<td>0.2</td>
<td>0.4</td>
<td>1</td>
<td>1.0192</td>
<td>15.4059 %</td>
</tr>
</tbody>
</table>

**Table 3: Case 2: increasing \( D \) and \( T \)**

<table>
<thead>
<tr>
<th>V</th>
<th>D</th>
<th>( r )</th>
<th>( \sigma_V )</th>
<th>( d_2 )</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>99</td>
<td>0.05</td>
<td>0.4</td>
<td>-0.0498</td>
<td>51.9888 %</td>
</tr>
<tr>
<td>100</td>
<td>99</td>
<td>0.05</td>
<td>0.4</td>
<td>-0.1564</td>
<td>56.2168 %</td>
</tr>
<tr>
<td>100</td>
<td>99</td>
<td>0.05</td>
<td>0.4</td>
<td>10</td>
<td>-0.2292</td>
</tr>
</tbody>
</table>

9
The variation of default probability depends on the variation of the variable $d_2$. We can resume the variation default probability in below:

- If $D_T$ increases/decreases then $DP$ will increase/decrease respectively.
- If $A_0$ increases/decreases then $DP$ will decrease/increase respectively.
- If $\sigma_A$ increases/decreases then $DP$ will increase/decrease respectively.
- If $T$ increases/decreases then $DP$ will increase/decrease respectively.
- If $r$ increases/decreases then $DP$ will decrease/increase respectively.

### 3.2 Risk-neutral default probability and Actual default probability

In order to obtain the default probability under the actual risk $Q$, we replace the neutral-risk interest rate value $r$ by actual interest rate value $\mu$.

$$Q(A_T < D_T) = N(-d_2\mu) = N(-d_2 - \rho_{A,M} S \sqrt{T})$$

where: $\rho_{A,M}$ is correlation of implicit asset return and stock market return, $S$ is Sharpe ratio.

The risk-neutral and actual default probability have concern with the risk-neutral expected return and actual expected return respectively. The risk-neutral expected return is a world where all investor/assets been risk-neutral. On the other side, the actual expected return is a real world. Evidently, the actual expected return is greater than the risk-neutral expected return, i.e., we argue that the investor does not want to invest for a risky asset if the expected return of this asset is less than the risk-neutral expected return.

The Girsanov (1960)'s theorem presented how to measure and to compare the risk-neutral default probability and actual default probability. According to this theorem, the risk-neutral default probability is greater than actual DP because the expected return $\mu$ greater than $r$ (Gray and Malone 2008).
4 The structural sovereign default risk model

4.1 Motivation

From the financial crisis in Thailand (1997) with the collapse of the Baht (3), especially many credit institutions went bankrupt. This collapse led to depreciation of the value of local currency of Korea, Indonesia. Through oil, Russia was the next victim of the financial crisis: Russian currency was devalued and Russia announced its default and denied to pay its government debt to creditors. Facing with this situation, Brazil raised interest rates to 40% in order to keep capital flow, but this scenario could not save Brazil out of the crisis (4). Because these countries held a very large amount of U.S. government bonds, the sovereign default of emerging countries led to the decline of the world economic system. Therefore, measuring the sovereign default probability is very essential.

The idea of transposing Merton’s model from the firm to the sovereign is to consider that the sovereign has two types of debts: a debt in local currency (for example, bills emitted by the central bank) and a debt in foreign currency. The government will always pay-off at first the debt in foreign currency, and the debt in local currency will only be paid-off if there is enough money.

Theoretically, the central bank will be able to create money to pay its government debts in local currency. However, this way would stimulate inflation and is opposed to the goal of stabilizing the economy. As a consequence, harmonizing between the debt repayment schedule and economic stability is a difficult task for macroeconomic policy-makers.

The inputs data in [Gray et al. (2007); Gray and Malone (2008)] were 5-year US Swap rates as the risk-free rate, 5-year maturity, monetary base, local currency debt and foreign currency debt for an empirical case of Brazil. The output of their model showed that the Brazil’s sovereign asset value arrives the default point in the Brazil's crisis period 2002-2003, and the Brazil’s risk indicator had a high correlation with its sovereign spread. This result illustrated clearly the real Brazilian economy.

[Souto et al. (2007)] used the volatility value of forward exchange rate as implied volatility from FX-option. The principal objective is to find the loan losses value of the main economic sectors for Turkey by means of using the CCA model. They created some scenarios to find the change of sovereign asset value and its volatility on condition that economic indicators shift, such as the change of exchange rate, risk-free interest rate or stock market index.

The purpose of this section, is to bring some extensions of the Merton model, advanced by [Gray and Malone (2008)]. That is a framework to compute the sovereign asset value and the sovereign asset volatility of emerging countries based on the option pricing theory by using information on sovereign liabilities.

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(3) Baht is the currency of Thailand
(4) see Friedman (1999)
4.2 Assumptions of the model

We consider an economy as a set of three interrelated sectors: the financial sector (banks), the non-financial (household and firms) and the sovereign sector (the combination between the government and monetary authority, also called the public sector). The extension of the Merton model is how to calculate the implicit sovereign asset value and its volatility.

The sovereign asset value is an unobservable variable because we cannot aggregate asset value of all components in the economy. In the actual market, only a few components can be observed; therefore, the aggregate value of all assets is not really easy to work with. The extension of Gray and Malone (2008) determined implicit sovereign asset and sovereign asset volatility by using the observed sovereign liabilities based on sovereign balance sheet.

There are three main assumptions in the extension of the Merton model:

- Assets follow a stochastic GBM process.
- The values of liabilities are derived from assets.
- Liabilities have a priority of debt: senior and junior. Foreign-currency debt is senior debt and this debt will be priority pay-off to creditors. Domestic-currency debt is junior debt that will be payed-off to creditors after the finishing payment of foreign-currency debt. For European countries (who use a common currency, the Euro), the debt is also usually in Euros, or strong currencies like the US Dollar, so it does not have a priority of debt. Therefore, this assumption imposes a limitation to the model when applied to European countries.

The sovereign balance sheet has two sides: assets and liabilities. The asset of monetary authorities include foreign reserves, credit to the government and others. The liabilities of monetary authorities are the monetary base, financial guarantees to the government, including guarantees to supply foreign currency to service the sovereign foreign-currency-denominated debt. The assets of the government are: net fiscal assets (including the seigniorage-tax inflation) and others. The liabilities of government are: credit to monetary authorities (including local currency debt held by the monetary authorities) and local currency debt held outside of the government and monetary authorities ... To simplify the model, we show the sovereign balance sheet:

<table>
<thead>
<tr>
<th>Sovereign Assets</th>
<th>Sovereign Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign-currency debt</td>
<td>Foreign-currency debt + Monetary base</td>
</tr>
</tbody>
</table>

Table 5: Sovereign balance sheet

Sovereign liabilities are combined by two factors: foreign-currency debt and sum of domestic currency debt and monetary base. Foreign-currency debt is the debt of the public sector in foreign currency held by foreigners. Domestic currency debt is the debt of the
public sector in domestic currency held by the private sector. The monetary base consists of
currency in circulation, bank reserves (required bank reserves, excess reserves, vault cash).
In banking, excess reserves are bank reserves in excess of the reserve requirement set by a
central bank. They are reserves of cash in excess of the required amounts. Changes in base
money correspond to changes in net foreign assets and net domestic assets. Governments
borrow by issuing securities, government bonds (long-term) and bills (short term) or borrow
directly from World Bank, IMF. The local-currency debt of the public sector is the debt
that held by private sector (Gray and Malone 2008). The foreign-currency debt of the
public sector is held usually primarily by foreigners. But according to Panizza (2008),
there are 2 approaches to determine external debt/internal debt. The first one is focused
on currency of debt issued, thus foreign-currency debt is considered external debt, and
local-currency debt is internal debt. But this approach has a problem. Actually, there are
some countries issued the foreign-currency debt in the internal market and issued domestic-
currency debt in the international market. The second one is focused on the residence of
the creditor, we can say that foreign-currency debt is debt from non-residents. Primarily,
the statistical information officially of external debt is based on the second approach.
In this section, we consider the foreign-currency debt that comes from by non-residents
and domestic-currency debt from the residents.
The sum of domestic currency debt and the monetary base is considered as "sovereign
equity (SE)" and foreign currency debt is considered as sovereign risky debt. We refer to
"foreign currency debt" as "risky debt" because the foreign debt is influenced by the ex-
change rate, i.e., more precisely, if the exchange rate between strong currency and domestic
currency increases, which means that the amount of foreign debt increases, and in a worst
case scenario, this country loses the ability to pay-off his foreign debt.

4.3 Definition

Sovereign default and default barrier

Sovereign default occurs when the values of sovereign assets fall below contractual lia-
tibilities also known as sovereign default barrier (named sovereign distress barrier or default
threshold).

At a sovereign level, the distress barrier approach is different than the one in the original
Merton model. In the original Merton model, the distress barrier is the firm’s total debt.
This means that a firm will default when the value of firm’s asset is below the value of total
debt. KMV model is based on the Merton model but with adjustments: the debt consists
of short-term obligations and long-term debt. A firm has more time to recover with respect
to the long-term debt. KMV’s research led it to conclude that the distress barrier is really
somewhere in between the short-term debt and the total debt. The default barrier is the
face value that will affect the pay-off in one year (Crosbie and R.Bohn 2001).

De Servigny and Renault (2007) used the ratio between long term foreign currency debt
($Debt_{LT}$) and short term foreign currency debt ($Debt_{ST}$) to define the default barrier. If

\[\frac{Debt_{LT}}{Debt_{ST}}\]
the ratio of long term foreign-currency debt divided by short term foreign-currency debt is less than 1.5 then default barrier = Debt_{ST} + 0.5 \times Debt_{LT} + one year interest payment. Otherwise, default barrier = Debt_{ST} + Debt_{LT} \times (0.7 - 0.3 \times Debt_{ST}/Debt_{LT}).

In many cases, the default barrier was defined by the sum of short term foreign debt plus a portion (varied from 0.5-0.8) of long term foreign debt. In this research, we define the default barrier as the sum of short term foreign currency debt plus a half long term foreign currency debt.

**Implicit sovereign asset and sovereign asset volatility**

The Gray et al.'s model calculates the implicit sovereign asset (SA) and the sovereign asset volatility based on the sovereign liabilities. Sovereign liabilities are the sum of domestic currency debt and the monetary base in foreign currency is called "sovereign equity" (SE), and foreign currency debt is considered risky sovereign debt.

Likewise with the credit risk model, the "sovereign equity" value can be modelled as an implicit Call option, and sovereign risky debt is modelled as the default-free value of debt minus an implicit Put option, by using equation (3) and (7).

**The first equation**

The call value for the SE is computed by:

\[ SE_S = SA_S N(d_1) - D_f e^{-r_f T} \times N(d_2) \] (10)

where \( d_1 = \frac{\ln\left(\frac{SA_S}{D_f}\right) + (r_f + 0.5\sigma^2_{SA})T}{\sigma_{SA} \sqrt{T}} \) and \( d_2 = d_1 - \sigma \sqrt{T} \); \( SA_S \) is the sovereign asset value, \( D_f \) is default barrier, \( r_f \) is risk-free foreign interest rate, \( \sigma_{SA} \) is sovereign asset volatility, \( T \) is maturity.

**The second equation**

We apply Itô’s lemma to call-option formula to derive a formula for the equity volatility:

(Ito’s lemma to calculate volatility of the process SE)

\[ SE \times \sigma_{SE,S} = N(d_1) \times \sigma_{SA} \times SA_S \] (11)

where \( d_1 = \frac{\ln\left(\frac{SA_S}{D_f}\right) + (r_f + 0.5\sigma^2_{SA})T}{\sigma_{SA} \sqrt{T}} \) and \( d_2 = d_1 - \sigma \sqrt{T} \); \( SA_S, \sigma_{SA} \) are sovereign asset value and sovereign asset volatility in foreign-currency.

One assumptions of Black & Scholes model is that there exists constant risk-free interest rate. But there is no clear market of risk-free interest rate. We have two possible of risk-free interest rates: the zero-coupon Treasury rates 10-year and swap rates. There are some recommendations of risk-free interest rate:

- **Zhu (2006)** used 5 year-US Swap rates as the risk-free rate and a 5-year time horizon for his analysis.
- Datastream recommended 3 month Treasury bills as risk-free interest rate.
- Bloomberg uses 10 year government bond rates as the risk-free interest rate.
In two main equations, there are two variables of risk-free interest rate: foreign interest rate and domestic interest rate. By default in this research, we define the foreign interest rate as a risk-free interest rate of US and domestic interest rate is risk-free interest rate of emerging countries.

We combine equations (10) and (11) to find the sovereign asset value $S_A$ and sovereign asset volatility $\sigma_{S_A}$ and then the sovereign default probability.

Probability of sovereign default under neutral risk is:

$$P = N(-d_{2r})$$

Probability of sovereign default under actual risk is:

$$Q = N(-d_{2\mu}) = N(-d_2 - \rho_{A,M} S\sqrt{T})$$

5 Empirical evidence from Argentina

5.1 Background of Argentina’s economy

Argentina’s crisis 1999-2002 and its debt default in 2001 were a serious crisis that damaged strongly the economy of this country. The overview of the Argentina economy can be considered in three factors: inflation rate, exchange rate and debt default. We can observe Figure 2(a) of inflation rate in the period 1975-2001: hyperinflation rate was 335% per year from 1975. Then, it maintained at the level of 10% and 20% per month during the next years, and peaked to 688% in 1984. Especially, in 1989, the inflation rate rocketed up to 3000%. By contrast, in 1997 it had dropped to 0.3%, and in 2001 it was negative (-1.5%).

Figure 2: Argentina’s annual inflation rate (%) and exchange rate

As shown in Figure 2(b), from 1997 to 2001, the exchange rate of Argentina currencies kept the regime of fixed exchange rate one-to-one Argentina peso-dollar (0.9995) but by early 2002 it jumped to 3%. This event marked the end of this regime. In 2002, a policy for
all banks was to convert all bank accounts denominated in dollars to pesos at the floating exchange rate. From 2001 to 2009, after finishing the regime of fixed exchange rate, the peso became devalued.

When reducing the budget deficit during the crisis 1999-2001, unfortunately, Argentina had to face failures repeatedly. So that it might burden a large foreign debt and the surged public debt displayed by a rapid increase of government debt. Particularly, the public debt ratio exceeded the allowed rate of 50% GDP in the end of 1999. In 2001, the amount of debt default in the total public debt went up to 80 billions $. And this number marked the event of Argentina’s default in this year (see in Figure 3 below).

![Figure 3: Argentina’s total defaulted debt (Source: Moody’s, * since 1983)](image)

5.2 Application CCA method to Argentina

5.2.1 Data sources

The data used in this study comes from many sources: The debt public in domestic currency is from MECON (Ministerio de Economia y production, Republic of Argentina), the external debt is taken from INDEC (National Institute of Statistic and Censuses, Argentina), Central Bank of Argentina (CBA).

5.2.2 Empirical results

In this study, we use an annual database for the period 1997-2009 for the following variables: interest rate 10-year of Government United States as the risk-free foreign interest rate, external debt, public debt monetary base in local currency and assume one year of maturity. We simply apply the official exchange rate to convert the value of the monetary base, public debt in local currency to foreign currency. For the default distress, we define the default barrier equal to the sum of external debt in the short term plus half of long term external debt. In order to find volatility of sovereign equity, we use the historical volatility method.

The main purpose in this section is to calculate the risk-neutral DP and the actual DP for Argentina by using the structural model of Gray et al. (2007), Gray and Malone (2008) and the Capital Asset Pricing Model (CAPM) respectively. Following these results, we show the evolution of two curves of DP with Argentina’s situation. In addition, we verify these findings with the Girsanov’s theorem.
Figure 4: Argentina’s input data

Source: MECON, INDEC, CBA
The risk-neutral default probability is applied by the structural model of Gray et al. (2007); Gray and Malone (2008). The outputs also are the sovereign asset value and its volatility.

The actual default probability is calculated by the Capital Asset Pricing Model (CAPM). We recall equation (9):

\[ N(-d_2) = N(-d_2 - \rho_{A,M} \sqrt{T}) \]

In order to find two values of \( \rho_{A,M} \) and Sharpe ratio\(^6\) we apply the Capital Asset Pricing Model (CAPM) to find \( \rho_{A,M} \).

\[ E(R_i) - r_d = \beta_i (E(R_m) - r_d) \]

Where: \( E(R_i) \) is the expected return on the Argentina’s sovereign asset, \( r_d \) is Argentina’s yield government bond, \( \beta_i \) is the sensitivity of the expected excess sovereign asset return to the expected excess market return, \( E(R_m) \) is expected return on the Argentina stock market.

We propose the Merval Index\(^7\) as the Argentina’s stock market index. By using the Merval index return and Argentina’s sovereign asset return, we obtain the correlation value between the Merval index return and sovereign asset return.

The results obtained can see in Figure 5: the line graph shows figures for the comparison between the Argentina’s sovereign asset and its default barrier, the sovereign asset (left axis) and its volatility (right axis), risk-neutral default probability and actual default probability.

As shown in Figure 5, The first graph shows the evolution between the Argentina’s sovereign asset and its default barrier between the years 1997 and 2009. From 1997 to 2001, they had a noticeable decrease. Furthermore, the distance between the two lines in this period is more and more smaller. This means that the Argentina’s default probability had a slight increase. This was explained by that the more the ratio of Argentina’s sovereign asset to its default barrier increased, the more Argentina’s default probability increased. At the end of 2001, the Argentina’s sovereign asset reached to the distress barrier, i.e., in that year, Argentina had a default of 80 billion USD of external debt. In 2005, the ratio of Argentina’s sovereign asset to its distress barrier was 1.3 while in 2008, this ratio was 1.7. Therefore, from 2002, there was a raise of the distance of the Argentina’s sovereign asset and its default barrier. This reveals a decrease of the Argentina’s default probability.

The second one is an important output that illustrates Argentina’s sovereign asset and degree distribution of its volatility. These results suggest the sovereign asset tended to decline from the beginning of 1998 to 2001 while Argentina’s assets volatility rose in 2001 that is exactly the period where Argentina’s default occurred.

The last one compares Argentina’s risk-neutral default probability and actual default probability. These plots confirm the Girsanov (1960)’s theorem that the risk-neutral default probability is greater than the actual one. The evolution showed the DP go up strongly

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\(^6\)Sharpe ratio = Average of excess return/Standard Deviation of excess return

\(^7\)Merval Index is the most important index of the Buenos Aires Stock Exchange
Figure 5: Argentina’s sovereign asset, its distress barrier, its volatility and its default probability

Source: author’s calculation. Notes: The sovereign asset corresponds to left axis and its volatility corresponds to right axis.
until 2001 and go down in post-period of 2001. In fact, this decline is thankful to Argentina improved policies of the post-crisis.

Regarding the lines graph between the sovereign default probability in Figure 5 and the Argentina’s inflation rate in Figure 4, these results also indicate both have the same evolution with Argentina’s economic situation. Therefore, we conclude the default probability obtained from the model which very homogeneous with Argentinian economy.

6 Conclusion

This paper shows the overview of the option pricing: Put and Call of the European option and its application for the credit risk model in order to calculate the default probability. The first objective of this paper is to demonstrate and to show the Merton’s model by re-establishing the Ito’s lemma and the option pricing theory. According to the Merton’s model, the default occurs when value of asset falls below the face value of all debt at maturity. Applying the option pricing theory, the difference between the asset and debt is the equity value that can be interpreted exactly as a Call option. From equation of Call option, we can find the default probability.

Based on the credit risk model and how to apply the Call option to calculate the default probability, we opted the idea of transposing Merton’s model from the firm to the sovereign by Gray et al. (2007); Gray and Malone (2008). The hypothesis is to consider that the sovereign has two types of debts: a debt in local currency and one in foreign currency. The central variable is the sovereign equity that is the sum of the monetary base plus the domestic debt. Likewise, we could interpreted the sovereign equity as a Call European option, and we applied to calculate the sovereign asset, its volatility and its default probability.

The main purpose was to verify a case study of Argentina’s default in 2002. Our results show the Argentina’s default probability tended to accelerate and reach a top in 2002 and decelerate after 2002. This mean that it is consistent with the Argentina’s situation. Conclusively, this contribution adds an empirical result of the structure model based on the extensions of Gray et al. (2007); Gray and Malone (2008) which have confirmed exactly with the real Argentinian economy.
References


